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Parameterized Approximation for Robust Clustering in Discrete Geometric Spaces

We consider the well-studied Robust (k, z)-Clustering problem, which generalizes the classic k-Median, k-Means, and k-Centre problems. Given a constant $z \geq 1$, the input to Robust (k, z)-Clustering is a set P of n weighted points in a metric space (M, δ) and a positive integer k. Further, each point belongs to one (or more) of the m many different groups $S_1, S_2, \dots, S_m$. Our goal is to find a set X of k centres such that $\max_{i \in [m]} \{\sum_{p \in S_i} w(p)\delta(p, X)^z\}$ is minimized. This problem arises in the domains of robust optimization [Anthony, Goyal, Gupta, Nagarajan, Math. Oper. Res. 2010] and in algorithmic fairness, for which a tight (under GAP-ETH) $(3^z + \epsilon)$-approximation algorithm exists [Goyal, Jaiswal, Inf. Proc. Letters, 2023].

Motivated by the tight lower bounds for general discrete metrics, we focus on geometric spaces such as the (discrete) high-dimensional Euclidean setting and metrics of low doubling dimension, which play an important role in data analysis applications. First, for a universal constant $\eta_0 > 0.0006$, we devise a $3^z(1 - \eta_0)$-factor FPT approximation algorithm for discrete high-dimensional Euclidean spaces thereby bypassing the lower bound for general metrics. We complement this result by showing that even the special case of k-Centre in dimension $\Theta(\log n)$ is $(\sqrt{3/2} - o(1))$-hard to approximate for FPT algorithms. Finally, we complete the FPT approximation landscape by designing an FPT $(1 + \epsilon)$-approximation scheme (EPAS) for the metric of sub-logarithmic doubling dimension.