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On the Hadamard-Fischer Inequality, the Inclusion-Exclusion Formula, and Bipartite Graphs

The classical Hadamard-Fischer-Koteljanskii inequality is an inequality between principal minors of positive definite matrices. In this work, we present an extension of the Hadamard-Fischer-Koteljanskii inequality, that is inspired by the inclusion-exclusion formula for sets. We formulate necessary and sufficient conditions for the inequality to hold. We describe general structures of the collection of index sets involved. In analyzing these structures, a graph-theoretical property that applies to bipartite graphs is found. We establish that if the vertices of a bipartite graph satisfy simple conditions, then the bipartite graph contains a vertex subgraph which is a cycle or a complete subgraph missing a matching. This result is reminiscent of the Hall's marriage theorem for bipartite graphs.