
HIMANSHU GUPTA, University of Regina

Matrix positivity preservers over finite fields

We resolve an algebraic version of Schoenberg's celebrated theorem characterizing the functions f with the property that the matrix $(f(a_{ij}))$ is positive definite for any positive definite matrix (a_{ij}) . Compared to the classical real and complex settings, we consider matrices with entries in a finite field. Here, we say that such a matrix is positive definite if all its leading principal minors are non-zero quadratic residues. We obtain a complete characterization of entrywise positivity preservers in that setting for matrices of a fixed dimension. When the dimension of the matrices is at least 3, we prove that, surprisingly, the positivity preservers are precisely the positive multiples of the field's automorphisms. We also provide a new connection between entrywise positivity preservers and automorphisms of Paley graphs. This is joint work with Dominique Guillot (University of Delaware) and Prateek Kumar Vishwakarma (Indian Institute of Science). The talk is based on the paper: <https://arxiv.org/abs/2404.00222>.