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Estimating the minimum positive eigenvalue of PSD matrices

An extensive body of literature addresses the estimation of eigenvalues of the sum of two symmetric matrices, $P+Q$, in relation to the eigenvalues of P and Q. Recently, we introduced two novel lower bounds on the minimum eigenvalue, $\lambda_{\min}(P+Q)$, under the conditions that matrices P and Q are symmetric positive semi-definite (PSD) and their sum $P+Q$ is non-singular. These bounds rely on the Friedrichs angle between the range spaces of matrices P and Q, which are denoted by $\mathcal{R}(P)$ and $\mathcal{R}(Q)$, respectively. In addition, both results led to the derivation of several new lower bounds on the minimum singular value of full-rank matrices. We extend these insights to estimate the minimum positive eigenvalue of $P+Q$, $\lambda_{\min}(P+Q)$, even if $P+Q$ is singular, in terms of the minimum positive eigenvalues of P and Q, namely $\lambda_{\min}(P)$ and $\lambda_{\min}(Q)$. Our approach leverages angles between specific subspaces of $\mathcal{R}(P)$ and $\mathcal{R}(Q)$, meticulously chosen to yield a positive lower bound. Additionally, we illustrate these concepts through relevant examples.