FLORIAN SCHWARZ, University of Calgary
The Lie Algebra of a group object

The tangent space of a Lie Group is a Lie Algebra. This is one of the most important basic results of differential geometry, sparking the entire field of Lie theory with its extensive applications to Physics.

Using group objects we will investigate the generalisation of this theorem in the broader setting of Cartesian tangent categories, encompassing among other things classical differential geometry, algebraic geometry and synthetic differential geometry. Tangent categories have been used to generalize various constructions from differential geomtetry, like connections, De Rham cohomology and differential equations.

We will see that (if a certain pullback exists) various results about Lie groups hold for group objects in Cartesian tangent categories. In particular the tangent bundle is trivial and a negation exist for the addition of tangent vectors, even if it was not a priori required to exist. This then allows us to define an external Lie-Algebra structure on the tangent space, generalizing a Lie group's Lie Algebra.