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The Erdős-Ko-Rado Theorem for semidirect products of transitive groups

A set of permutations $\mathcal{F}$ of a finite transitive group $G \leq \mathrm{Sym}(\Omega)$ is *intersecting* if any two permutations in $\mathcal{F}$ agree on some elements of Ω. An Erdős-Ko-Rado (EKR) type theorem for the transitive group G in this context gives the size and the structure of the largest intersecting sets.

In 2015, Ahmadi and Meagher asked whether it is possible to give an EKR type theorem for the semidirect product $G \rtimes \mathbb{Z}_2 \leq \mathrm{Sym}(\Omega)$, provided that we have a "nice enough" EKR theorem for the transitive group $G \leq \mathrm{Sym}(\Omega)$. There is no general answer to this question and the structure of the largest intersecting sets vastly depends on the action of G.

In this talk, I will focus on an example of semidirect product with cyclic groups for which the largest intersecting sets are much more complex. In particular, I will talk about the largest intersecting sets for the actions of the general linear group $\mathrm{GL}_2(q)$ and the general semilinear group $\Gamma\mathrm{L}_2(q) = \mathrm{GL}_2(q) \rtimes \mathrm{Aut}(\mathbb{F}_q)$ on non-zero vectors of $\mathbb{F}_q^2$. Note that if p is a prime, then $\Gamma\mathrm{L}_2(p^2) = \mathrm{GL}_2(p^2) \rtimes \mathbb{Z}_2$. In contrast to $\mathrm{GL}_2(q)$ which only has two classes of largest intersecting sets, the group $\Gamma\mathrm{L}_2(q)$ has multiple classes of intersecting sets, and they need not be subgroups.