CH-Thirty Years Later
CH - trente ans plus tard
(Org: **Xiangke Chang** (AMSS, Institute of Mathematics, Chinese Academy of Sciences) and/et **Jacek Szmigielski** (University of Saskatchewan))

STEPHEN ANCO, Brock University
Peakons: some simple questions with unexpected answers

Peakons are peaked travelling waves which arise as solutions of the integrable Camassa-Holm equation in water wave theory discovered 30 years ago. In the explosion of work on peakons following that discovery, several basic questions have been asked about the nature of peakons —

What is the most general class of nonlinear dispersive wave equations possessing peakon solutions?

Is integrability necessary for existence of multi-peakon solutions?

Are peakons best understood as weak solutions or distributional solutions?

Does the NLS equation have a peakon counterpart?

How to find integrable peakon equations systematically?

In this talk, I will review some of my contributions to understanding and attempting to answer these questions over the past decade, which have led to some unexpected and on-going new developments.

ROBERTO CAMASSA,
Mathematical modeling of shallow water wave propagation.

Arguably, the mathematical modeling of water waves has given rise to some of the most interesting advances in the study of nonlinear partial differential equations. This talk will revisit some of these models and their derivation from first principle physics, focussing on some results that are possibly less well known, and consider recent extensions of existing models that can cover a wider range of wave phenomena.

MICHAEL GEKHTMAN, University of Notre Damw
Integrable systems and cluster algebras

We review several constructions of integrable systems with an underlying cluster algebra structure, in particular the Gekhtman-Shapiro-Tabachnikov-Vainshtein construction based on perfect networks and the Goncharov-Kenyon approach based on the dimer model. Based on a survey joint with A. Izosimov.

KATRIN GRUNERT, Norwegian University of Science and Technology
Uniqueness for the Camassa-Holm equation

Weak solutions of the Camassa–Holm equation might not be unique due to various admissible prolongations beyond wave breaking. The two most prominent continuations, whose existence can been established with the help of a generalized method of characteristics, are called conservative and dissipative. Here, we will discuss the uniqueness for these solution concepts by establishing a bijection between the properties specific for each solution type and the corresponding solution operator defined via a generalized method of characteristics.

ALEX HIMONAS, University of Notre Dame
Analysis of the CH equation and family

In this talk we shall examine the various facets of the analysis of the Camassa-Holm (CH) equation and a related family of equations. Our focus will be on the well-posedness and ill-posedness of their initial value problem. In particular, we shall examine a number of interesting properties that they possess (like peakons) and influence their analysis. The talk is based on work with C. Holliman, C. Kenig, G. Misiolek, G. Petronilho, and G. Ponce.

HELGE HOLDEN, Norwegian University of Science and Technology
On the stochastic Camassa—Holm equation with transport noise

We will discuss recent work regarding the stochastic Camassa—Holm equation $u_t + uu_x + P_x + \sigma u_x \circ dW = 0$ and $P - P_{xx} = u^2 + u_x^2/2$. În particular, we will show existence of a weak, global, dissipative solution of the Cauchy initial-value problem on the torus. This is joint work with L. Galimberti (King's College), K.H. Karlsen (Oslo), and P.H.C. Pang (NTNU/Oslo).

DARRYL HOLM, Imperial College London
Emergent singular solutions (ESS) in nonlinear wave PDEs

We discuss emergent singular solutions (ESS) in nonlinear wave PDEs.

(1) Start with asymptotic expansion for 1D shallow water waves.

(2) Identify the b-equation in n dimensions, H-Staley [2003]

Integrable in 1D, $b = 2$ Camassa-H [1993], $b = 3$ Degasperis-Processi [2002]

(3) Q: Why is $b = 2$ special? Is ESS is a property of integrability?

A: No. The ESS solution Ansatz is a momentum map, H-Marsden [2005]

(4) Are there other geodesic ESS with $b = 2$ in 1D? Yes! Fringer-H [2001]

(5) ESS for Stochastic CH? Yes! Crisan-H [2019] and Bendall-Cotter-H [2022]

(6) Are there ESS for $b = 2$ and $W^{1,r}$ norm? Cotter-H-Pryer [2023]

(7) Are there ESS embeddings for PDEs in 2D and 3D?. H-Staley [2004]

ANDREW HONE, University of Kent
An elliptic analogue of the Camassa-Holm equation

In this talk, an elliptic analogue of the Camassa-Holm equation is described, which was obtained in a classification of nonlocal CH-type equations with a 3rd order local symmetry. The equation contains not only nonlocal terms (like CH), but also coefficients that are implicitly defined in terms of an ordinary differential equation that is solved in elliptic functions. The complete structure and properties of this equation are still somewhat mysterious, so here we present some preliminary observations and describe various open problems. This is based on joint work with Ben Gormley and Vladimir Novikov.

HANS LUNDMARK, Linköping University, Sweden
Dynamics of peakons and antipeakons in Novikov's equation

Novikov's equation is a cubically nonlinear integrable PDE of Camassa–Holm type. It admits globally defined conservative peakon–antipeakon solutions similar to those of the Camassa–Holm equation, but with a much richer variety of possible behaviours, as will be explained and illustrated in this talk.

VLADIMIR NOVIKOV, Loughborough University
Towards the complete classification of integrable Camassa-Holm type equations

After 30 years of discovery of the Camassa-Holm equation the complete classification of integrable equations of this type remains an open problem. Camassa-Holm type equations can be viewed as negative flows of hierarchies of integrable evolutionary partial differential equations. There are various approaches to tackle integrability of negative flows. In this talk I will review various integrability tests applicable to such type systems and present the classification results. Some of the thus found equations are new.

DMITRY PELINOVSKY, McMaster University

Traveling waves in the Camassa-Holm equations: their stability and instability

The Camassa–Holm equation in one spatial dimension admits traveling solitary and periodic waves with the smooth, peaked, and cusped profiles. I will overview recent results on the stability analysis of the traveling solitary waves in the Camassa–Holm equation and its extensions, the b-family of the Camassa–Holm equations. In particular, we proved the spectral and orbital stability of traveling waves with the smooth profiles. At the same time, we showed that the traveling waves with the peaked profile are linearly and nonlinearly unstable in $H^1 \cap W^{1,\infty}$ despite their orbital stability in H^1. More recently, we proved the transverse stability of one-dimensional solitary waves with the smooth profiles in the two-dimensional generalization of the Camassa–Holm equation.

ZHIJUN QIAO, NCST/UTRGV

Integrable CH hierarchy and beyond

In this talk, we will present one of the CH developments, namely, the Camassa-Holm hierarchy and its integrable structure etc. We will see how the CH hierarchy is related to finite-dimensional integrable systems, and furthermore algebro-geometric solutions of the CH hierarchy are shown on a symplectic submanifold. Other similar peakons models, including the DP, the b-family, and cubic equations (MOCH, FORQ/MCH, Novikov etc) will be mentioned as well. Some results are selected from my 2003 CMP paper, but we will present our recent studies on the higher-order CH type equations as well.

BO XUE, Zhengzhou University, China