Student Research Talks

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COURTNEY ALLEN, University of Guelph

A de novo implementation of the Anaerobic Digestion Model 1 raises questions about computational speed

The Anaerobic Digestion Model 1 is the quasi industry standard for modelling anaerobic digestion. It was conceived as a system of 35 ordinary differential equations (ODEs), but an alternate form was developed to reduce the stiffness of the system and therefore improve computation time. This alternate form is a system of differential algebraic equations (DAEs). The form of ADM1 (ODE vs DAE) is assumed to be the limiting factor when it comes to computation time. However, comparing a *de novo* ODE implementation written in Julia against existing DAE implementations in Python and Java shows that the Julia ODE implementation outperforms the DAE implementations. This result indicates that computational speed depends more on the numerical methods used to solve the system than the form of the system itself.

MANAL ALZAHRANI, University Of Ottawa

Computing the Faithful Dimension of Certain Classes of p-Groups via the Orbit Method

The *faithful dimension* of a finite group G over $\mathbb{C}$, denoted by $m_{\mathsf{faithful}}(G)$, is defined to be the smallest integer m such that G can be embedded in $GL_m(\mathbb{C})$. We are interested in computing the faithful dimension of a p-group of the form $G_q := \exp(\mathfrak{f}_{n,c} \otimes_{\mathbb{Z}} \mathbb{F}_q)$, where $q = p^f$ and $\mathfrak{f}_{n,c}$ is the free nilpotent $\mathbb{Z}$-Lie algebra of class c on n generators.

In 2019, Bardestani et al. expressed the faithful dimension of G_q as the solution to a rank minimization problem by applying Kirillov's orbit method. This approach is dependent on the concept of the *commutator matrix* associated to the nilpotent $\mathbb{Z}$-Lie algebra. As a result, they were able to compute the faithful dimension for nilpotency classes $c = 2$ and $c = 3$.

Following Bardestani et al. rank minimization method, we obtain the faithful dimension of the free nilpotent $\mathbb{Z}$-Lie algebra of class $c = 4$ on n generators. An explicit description of the commutator matrix is obtained by using the *Hall basis* of the free $\mathbb{Z}$-Lie algebra $\mathfrak{f}_{n,4}$.

We also explore the computation of the faithful dimension for nilpotency class $c = 5$. With the aid of computer-assisted symbolic computations, we obtain an upper bound for $m_{\mathsf{faithful}}(\exp(\mathfrak{f}_{n,5} \otimes_{\mathbb{Z}} \mathbb{F}_q))$ of magnitude $n^5 q^4$.

ANKANA DEY, Université de Sherbrooke

Metacommunity Theory : Adapting for the Human Microbiome

The human microbiota is composed of a diversity of bacteria, fungi, protists and viruses and it is dynamic; it changes based on what we eat, the medication we take or diseases we may have. In this respect, it is interesting to study how microbial species disperse and change across different parts of our body. To approach this problem, we relied on metacommunity theory, which was originally developed to study macroecological systems but which has been suggested to study microbiomes. In our project we proposed to adapt metacommunity theory for the particularity of humans' physiological system by developing a multivariate Lotka-Volterra Competition Diffusion model, which, is also a continuous extension of metacommunity theory. Moreover, we make our mathematical model more realistic by also accounting for migration, i.e. the arrival of new microbial species within the system. The parameters of this new theoretical model are estimated through Bayesian modelling using data from the Human Microbiome Project. In this presentation, the mathematical model and its associated statistical model will be presented, which relied on Markov chain Monte Carlo.

ARIAN HAGHPARAST, York University

Critical probability for phase transition in a degenerate random environment

Percolation is a well-studied phenomenon in statistical physics and probability theory, which describes the behavior of fluids, gases, or other substances as they pass through a random environment. The concept of percolation theory has also been extended to other fields, including computer science, network analysis, and ecology. The critical point is a fundamental concept in percolation theory, which refers to the point at which the system undergoes a phase transition from one state to another.

This study explores how an agent behaves in a randomly generated 2D environment. The cells of our grid environment are randomly filled with either $\uparrow$ or $\leftarrow\updownarrow\rightarrow$ arrows which determine the available adjacent cells, and a parameter p controls the frequency of each one. We are interested in simulating the agent's behavior and, more importantly, approximating a value for p that acts as a critical point for our system and causes a significant change in the behavior of the environment. We use computer simulation to investigate and determine the critical probability value

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DIBA HEYDARY, University of Toronto
Adventures in Geometric Topology: An Introduction to the Mapping Class Group

The mapping class group is one of the key algebraic invariants of a topological space, and a chief tool for studying the automorphism group of a manifold. Frequently omitted from undergraduate curriculum, mapping class groups are an essential component of geometric topology, and geometric group theory. This will be an expository (and pictorial!) talk introducing students to these ideas, including their wider role in hyperbolic geometry and the study of moduli spaces. Familiarity with topology at the level of Munkres' "Topology" will be assumed.

ACHINTYA RAYA POLAVARAPU, University of Alberta
Stonean representation of sup-completion of a vector lattice

The study of vector lattices and their relationship with stochastic processes has been an active area of research in recent years. The concept of sup-completion is a powerful tool in this field due to its properties of extending the notion of supremum to partially ordered sets that may not have a natural upper bound. In this talk, we will briefly introduce the field of vector lattices and provide a representation of the sup-completion using the Maeda-Ogasawara theorem. This representation will essentially reduce the sup-completion to studying the properties of continuous functions on the Stone space of the vector lattice. Joint work with Vladimir Troitsky.

ZHEN SHUANG, Memorial Univerisity of Newfoundland
Weighted p-Laplacian Parabolic Equation and Signal Decomposition

We show the existence of solutions for a new kind of weighted p-Laplacian parabolic equation and its applications in signal processing in which a signal is decomposed into four parts. The presence of solutions is proved by the Faedo-Galerkin method. The spectrum and decomposition of a signal are constructed through numerical methods in Matlab. Fractional order p-Laplacian and fractional order derivatives are expressed explicitly in the introduced model, so it is easy to implement in Matlab.

SILAS VRIEND, McMaster University
On a Free-Endpoint Isoperimetric Problem in $\mathbb{R}^2$

Inspired by a mixed planar partitioning problem, we investigate using classical techniques what can be said of the existence, uniqueness, and regularity of minimizers in a certain free-endpoint isoperimetric problem. By restricting to curves which are expressible as graphs of functions, we prove a full existence-uniqueness-regularity result using a convexity technique inspired by work of Talenti. The problem studied here can be interpreted physically as the identification of the equilibrium shape of a sessile liquid drop in half-space (in the absence of gravity). This is a well-studied variational problem whose full resolution requires the use of geometric measure theory, in particular the theory of sets of finite perimeter, but here we use a more direct, classical geometrical approach. We present conjectures on other mixed planar partitioning problems throughout.

TONATIUH MATOS WIEDERHOLD, University of Toronto
Two-player infinite games on posets

Fix a subset of reals S. Alice and Bob take turns picking real numbers, with the only restriction that they must pick strictly between the previous two selected reals. Alice wins Baker's Game if she can legally pick an element of S after the game is over. Bob wins if he can prevent this.

If S is countable, then Bob has a winning strategy. (This is a game-theoretic proof that the reals are uncountable.) Matt Baker asked if the converse is true. Recently, Brian and Clontz used elementary submodels to give a positive answer to this question.

In this talk, we will present some variations of Baker's Game and recent contributions. We mainly discuss generalizations for which having winning strategies characterizes interesting properties of the posets on which the game is played. We also mention connections to Banach-Mazur, Hausdorff gaps, the perfect set property, partition problems, and some nice open questions.

This is joint work with Luciano Salvetti.

CHENGJUN YUE, Memorial University of Newfoundland
Three Diffusion-wave Models with Nonlocal Operators for Image Denoising

Images are easy to be contaminated by noise, a kind of high-frequency component, in formation, recording, transmission and etc. The process to remove noise from a noisy image, so as to restore the true image is referred to as image denoising. To be precise, we have the model

$$u_0 = u + n,$$

where u_0 is noisy image, u is clear image, and n is the additive noise. Our target is to recover u from u_0.

We study three models for image denoising. Our models are based on the diffusion equation and wave equation. Traditionally, local operators are often applied in PDE-based models. In our study, we exploit the nonlocal operators, Riesz potentials and fractional Laplacian, to our models. The numerical results present an improvement in the denoising effect compared with the heat equation.

BEHNOOSH ZAMANLOOY, McMaster University
Strong Data Processing Inequalities for Locally Differentially Private Mechanisms

We investigate the strong data processing inequalities of locally differentially private mechanisms under a specific f-divergence, namely the E_γ-divergence. More specifically, we characterize an upper bound on the E_γ-divergence between $P\mathsf{K}$ and $Q\mathsf{K}$, the output distributions of an ε-LDP mechanism K, in terms of the E_γ-divergence between the corresponding input distributions P and Q. Interestingly, the tightest such upper bound in the binary case turns out to have a non-multiplicative form. We then extend our results to derive a tight upper bound for general f-divergences. As an application of our main findings, we derive a lower bound on the locally private Bayesian estimation risk that is tighter than the available divergence-based bound in the literature.