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Generators and relations for 3-qubit Clifford+CS operators

We give a presentation by generators and relations of the group of 3-qubit Clifford+CS operators. The proof roughly consists of two parts: (1) applying the Reidemeister-Schreier theorem repeatedly to an earlier result of ours; and (2) the simplification of thousands of relations into 17 relations. Both (1) and (2) have been formally verified in the proof assistant Agda. The Reidemeister-Schreier theorem gives a constructive method for computing a presentation of a sub-monoid given a presentation of the super-monoid. To achieve (2), we devise an almost-normal form for Clifford+CS operators. Along the way, we also identify several interesting structures within the Clifford+CS group. Specifically, we identify three different finite subgroups for whose elements we can give unique normal forms. We show that the 3-qubit Clifford+CS group, which is of course infinite, is the amalgamated product of these three finite subgroups. This result is analogous to the fact that the 1-qubit Clifford+T group is an amalgamated product of two finite subgroups.