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High Order Finite Difference Methods for Interface Problems

Interface problem arises in many applications such as modeling of underground waste disposal, oil reservoir, composite material, and many others. The coefficient a, the source term f, the wave number k, the solution u, and the flux $a\nabla u \cdot \vec{n}$ are possibly discontinuous across the interface curve Γ in such problems. To obtain the reasonable numerical solution, the higher order numerical scheme is desirable. Firstly, we propose a sixth order compact 9-point finite difference method (FDM) for the Poisson interface problem with the singular source. For the elliptic interface problem with the discontinuous and piecewise smooth coefficient, we propose a high order compact 9-point FDM and a high order local calculation for the approximation of the solution u and the gradient ∇u respectively. Furthermore, we derive the compact 9-point FDM with high accuracy order and/or M-matrix property for the elliptic cross-interface problem. Finally, we propose a sixth order compact FDM for the Helmholtz Equation with the singular source. To reduce the pollution effect, we propose a new pollution minimization strategy that is based on the average truncation error of plane waves. All above schemes are developed on the uniform Cartesian grid in a rectangular domain. Our numerical experiments confirm the flexibility and the expected order accuracy in l_2 and l_∞ norms of the proposed schemes. Except the Helmholtz Equation, we prove the corresponding convergence rate for the proposed schemes using the discrete maximum principle. This is joint work with Bin Han, Peter Minev and Michelle Michelle.