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Error Assessment for a Finite Element - Neural Network Approach Applied to Parametric PDEs

A parametric PDE

$$\mathcal{F}(u(x;\mu);\mu) = 0 \quad x \in \Omega, \quad \mu \in \mathcal{P}, \tag{1}$$

where Ω denotes the physical domain and $\mathcal{P}$ the parameters domain, is considered. Depending on the differential operator $\mathcal{F}$, numerical methods used to approximate the solution of (1) may be time consuming, which is an issue in the many query context (e.g. if an inverse problem has to be solved) or if the solution is needed in real time. To overcome this issue, we aim to build a neural network that approximates the map $(x;\mu) \mapsto u(x;\mu)$. For this purpose, we compute numerical approximations $u_h(x;\mu_i)$ of $u(x;\mu_i)$, $i = 1, \ldots, M$, and use them as a train set for the neural network. We are then interested in the error between u and the approximation given by the network, that we denote by $u_{\mathcal{N}}$. More precisely, we want to estimate $||u - u_{\mathcal{N}}||_{L^2(\Omega\times\mathcal{P})}$, which can be split as

$$||u - u_{\mathcal{N}}||_{L^2(\Omega\times\mathcal{P})} \leq ||u - u_h||_{L^2(\Omega\times\mathcal{P})} + ||u_h - u_{\mathcal{N}}||_{L^2(\Omega\times\mathcal{P})}.$$

In the presentation, we discuss how the two error terms can be estimated, to what extend we can ensure that they are balanced and we present numerical results for a model problem. If time permits, we will also discuss how the neural network can be used to solve an inverse problem.