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Wendel's theorem and the neighborliness of random polytopes

A convex polytope is k -neighborly if every subset of at most k vertices is a face of the polytope. A well-known feature of random polytopes in high dimension is that they often have a surprisingly high degree of neighborliness. For example, it is known that Gaussian random polytopes are cd -neighborly w.h.p. for some constant $1 > c > 0$ when the number of vertices is proportional to the dimension. Furthermore, work of Donoho and Tanner and Vershik and Sporyshev shows that there is a threshold for the neighborliness of Gaussian random polytopes. We show that a similar thing happens when the vertices are i.i.d. according to an arbitrary absolutely continuous probability distribution on \mathbb{R}^d . As a concrete example, our result implies that if for each d in \mathbb{N} we choose an arbitrary absolutely continuous probability distribution μ_d on \mathbb{R}^d and then set P to be the convex hull of an i.i.d. sample of at most $n = 10d/9$ random points from μ_d , the probability that P is $(d/10)$ -neighborly approaches one as $d \rightarrow \infty$. We will also give an example of a family of distributions which show that this result is close to best possible. The proof relies on a generalization of Wendel's theorem due to Wagner and Welzl.

This material is based upon work supported by the National Science Foundation under Grants CCF-1657939, CCF-1934568 and CCF-2006994.