Interplay Between Analysis and Convexity
Interaction entre l'analyse et la convexité

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WANJUN AI, Southwest University

A Geometric Constructive Proof for the 2D Discrete Minkowski Problem

The 2-dimensional discrete Minkowski problem seeks to determine the necessary and sufficient conditions for the existence of a polygon in R^2 with n facets, whose outer unit normals are $u_1, u_2, \ldots, u_n \in S^1$ and such that the facet whose outer unit normal is u_i has length a_i, where $a_1, a_2, \ldots, a_n > 0$. Minkowski solved this problem in 1897 using a variational argument. In this talk, we will present a geometric constructive proof based on special reflections, which offers new insights into the problem and proposes the study of a new type of flow on 2-dimensional polygons.

WEN AI, Memorial University of Newfoundland

The L_p dual Minkowski problem for unbounded closed convex sets

The Brunn-Minkowski theory for bounded closed convex sets is the core in convex geometry, especially the study on the Minkowski problem. From the classical Minkowski problem to the recent L_p dual Minkowski problem, the past century has witnessed the great development on the Minkowski type problems. The significance of the Minkowski type problems can be revealed in other areas, for instance, differential geometry and PDEs. The unbounded closed convex sets have proved to be important in differential geometry, PDEs, singularity theory and commutative algebra. This triggers the study of the corresponding geometric theory for unbounded closed convex sets, with particular interest on the Minkowski type problems.

In this talk, I will talk about my recent work on the L_p dual Brunn-Minkowski theory for unbounded closed convex sets. In particular, I will explain the (p, q)-th dual curvature measure for unbounded closed convex sets, and present an existence and uniqueness of solution to such L_p dual Minkowski problem in the unbounded setting.

KÁROLY BEZDEK, University of Calgary

The Kneser-Poulsen conjecture for uniform contractions revisited

The Kneser-Poulsen conjecture (1955) states that if a finite set of (not necessarily congruent) balls in the Euclidean d-space is rearranged so that the distance between each pair of centers does not increase, then the area of the union does not increase, and the area of the intersection does not decrease. This was proved for $d = 2$ by K. Bezdek and R. Connelly in 2002. The Kneser-Poulsen conjecture is still open for all $d > 2$. Consider the following special case. Take finitely many congruent balls in the Euclidean d-space and reposition them (without changing their radius) by applying a uniform contraction to their centers. Here a uniform contraction maps the first set of centers onto the second set of centers such that the pairwise distances in the first set of centers are larger than or equal to all pairwise distances in the second set of centers. The lecture surveys the progress towards a proof of the Kneser-Poulsen conjecture for uniform contractions of congruent balls.

ALMUT BURCHARD, University of Toronto

On pointwise montonicity of heat kernels

The fact that the heat kernel $K_t(x, y)$ on the standard sphere decreases with the distance between the points x and y has important consequences in Probability and Functional Analysis. In a recent paper, Alonso-Oran, Chamizo, Mas, and

Martinez asked, *What are the pointwise monotonicity properties of the heat kernel on a general Riemannian manifold?* [See arXiv:1807.11072, Section 1.] I will describe current work with Angel Martinez on metrics on compact manifolds for which the heat kernel decreases monotonically as y moves along a minimal geodesic emanating from x. We prove that such metrics are extremely rare, while also providing a new example.

MIN CHEN, McGill University
IN-HOMOGENEOUS GAUSS CURVATURE FLOWS

We consider the flow of convex hypersurfaces in Euclidean space R^{n+1} under the in-homogeneous speed functions of Gauss curvature. We establish the existence and convergence of the flow to a limit which is the round sphere (after rescaling) under appropriate conditions of the speed functions. This generalized the celebrated results on Gauss curvature flow by Andrews-Guan-Ni and Brendle-Choi-Daskalopoulos. This is joint work with Prof.Pengfei Guan and Jiuzhou Huang.

JOSHUA FLYNN, CRM/ISM, McGill University
Hardy Inequalities and Mean Convex Domains

In this talk we present on our recent results on sharp Hardy integral identities and inequalities where the weights are in terms of the distance function to the boundary of a domain. The identities are used to address the existence of minimizers for the corresponding inequalities on weakly mean convex domains. For less regular domains, we obtain Hardy identities and inequalities in terms of the mean distance function to the boundary. This is a joint work with N. Lam and G. Lu.

FERENC FODOR, University of Szeged, Hungary
A central limit theorem for the area of random disc-polygons

We consider the following probability model of random disc-polygons. Let K be a convex disc in the Euclidean plane with at least C^2_+ smooth boundary (twice continuously differentiable with everywhere positive curvature). Fix $r > 0$ such that it is larger than the maximum radius of curvature of the boundary of K. Take n independent random points from K according to the uniform probability distribution. Let K^r_n be the intersection of all radius r closed circular discs that contain the random points. This object is called a (uniform) random disc-polygon, and it is known to be contained in K. Various asymptotic properties of K^r_n (as $n \to \infty$) have been determined before, including an asymptotic formula for the expectation of the area of K not covered by K^r_n, and also lower and upper bounds of matching orders of magnitude (in n) for the variance of the area of K^r_n. In this talk we present a quantitative central limit theorem for the area of K^r_n based on Stein's method. Joint work with Dániel Papvári (Szeged, Hungary).

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JULIÁN HADDAD, Universidad de Sevilla
Higher-order Petty's projection inequality

In 1970 Schneider defined and studied the higher order difference body $D^\ell K \subseteq \mathbb{R}^{nm}$ of a convex body $K \subseteq \mathbb{R}^n$. Inspired by the connection of the difference body and the projection body through the covariogram function, we define the higher-order projection body and prove the corresponding Petty projection inequality. The result uses a volume-increasing fiber symmetrization method.

(joint work with D. LANGHARST, E. PUTTERMAN, M. ROYSDON AND D. YE)

FANG HONG, McGill University
Sharpened Minkowski Inequality in Cartan-Hadamard Spaces

Minkowski inequality describes the relationship between total mean curvature of a surface and its area. Extension of Minkowski inequality to hyperbolic space and finding the sharp inequality have been a long standing problem. We will discuss recent paper by M. Ghomi and J. Spruck, in which they generalized Minkowski inequality to general spaces with non-positive curvature via harmonic mean curvature flow. We will further discuss sharper inequality we get based on their results.

SERGII MYROSHNYCHENKO, Lakehead University
How far apart can centroids be?

The orthogonal projection of the centroid (barycenter, center of mass) of a convex body K onto a hyperplane H, and the centroid of projection of K onto H coincide if K is centrally-symmetric. In general, this is not the case for non-symmetric convex bodies. In this talk, we investigate how far apart these points can be with respect to the width in the direction of the segment connecting them. The optimizers are described as well. The talk is based on the joint work with K. Tatarko and V. Yaskin (https://arxiv.org/abs/2212.14456).

ZHEN SHUANG, Memorial University of Newfoundland
Weighted Laplacian Evolution Equation and Signal Decomposition

We show the existence of solutions for new types of weighted Laplacian wave equations and their applications in signal processing in which a signal is decomposed into four parts. The presence of solutions is proved by the Faedo-Galerkin method. The spectrum and decomposition of a signal are created through the discrete solutions of the equations in Matlab. Fractional order Laplacian and fractional order derivatives are expressed explicitly in the introduced equations, so it is easy to implement in Matlab.

BEATRICE-HELEN VRITSIOU, University of Alberta
The Illumination Conjecture for convex bodies with many symmetries

We will show how to verify the Hadwiger-Boltyanski Illumination Conjecture (along with its equality cases) for 1-symmetric convex bodies of all dimensions (that is, convex bodies with the symmetries of the cube) and some cases of 1-unconditional convex bodies as well (that is, convex bodies with the symmetries of a rectangular box).

This is joint work with Wen Rui Sun.

FANHENG XU, Memorial University
Geometric Sharp Sobolev-type Principle for The Graphic Submanifolds of Euclidean Space

I will present a recently established sharp Sobolev-type principle for a compact n-dimensional graphic submanifold (Σ, g) of $\mathbb{R}^{n+m}$. This principle was established using a positive smooth function f on Σ and the absolute value of the determinant of g. We demonstrate that the principle holds with equality when f is constant on Σ, $G = 1$ on $\partial\Sigma$, and Σ is a round ball in $\mathbb{R}^n$. Additionally, the inequality yields a sharp isoperimetric inequality for graphic submanifolds of Euclidean space with the unit metric determinant. This work was done in collaboration with Professor J. Xiao.

CHENGJUN YUE, Memorial University of Newfoundland
A cartoon+texture image decomposition based on interpolation spaces

Image decomposition is referred to separating a given image into multiple layers of components with different characteristics. That is an essential problem in image processing since usually there is a need for extracting or modifying specific geometric structures of an image before further analysis. We focus on the decomposition of image f

$$f = u + v$$

where u is a piecewise constant component and v is an oscillation component. When f is a smooth image contaminated by noise, it comes back to the image-denoising model.

One of the typical methods for achieving this target is the variational method. From the famous (BV, L^2) decomposition, inspired by the $(BV, B\dot{M}O^{\alpha})$ and $(BV, \dot{W}^{\alpha,p})$ decomposition, we establish a new model $(BV, \dot{W}^{\alpha,p,\infty})$ for image decomposition.

ZENGLE ZHANG, Chongqing University of Arts and Sciences
The (φ, ψ) Orlicz mixed affine and geominimal surface areas

The affine surface area is one of the central notions in the Brunn-Minkowski theory for convex bodies. Its special properties make the affine surface area very useful in the valuation theory, approximation of convex bodies by polytopes, affine isoperimetric inequalities, etc. The geominimal surface area is closely related to the affine surface area, and can be used to connect various geomtrices such as affine geometry, Minkowski geometry and relative geometry. It natrually leads to the fundamental object: Petty body. In this talk, we will present the (φ, ψ) Orlicz mixed affine and geominimal surface areas, and discuss their related properties, such as homogeneity, affine invariance, affine isoperimetric inequalities and continuity.

JIAZU ZHOU, Southwest University, China
Isoperimetric inequalities for mean curvature integrals

The classical isoperimetric problem asserts that the ball has the maximum volume among domains with the given surface area in the n-dimensional Euclidean space. The isoperimetric problem is equivalent to the isoperimetri inequality that contains the volume and the surface area of the domain. We try to study the isoperimetric inequalities for mean curvature integrals with the smooth boundary assumption for the domain in the n-dimensional Euclidean space.

XIA ZHOU, Memorial University of Newfoundland
On the optimal Orlicz norms and the general dual Musielak Orlicz-Minkowski problems

As one of the cornerstones of the classical Brunn-Minkowski theory for convex bodies, the Minkowski problem plays a crucial role not only in convex geometry, but also in other related fields, such as, differential geometry, PDEs and optimal transport.

In this talk, a new Minkowski-type problem will be introduced, which involves the homogeneous general dual volume. I will talk about how to derive the corresponding general dual Musielak-Orlicz curvature measures. The characterization problems to those measures, i.e., the general dual Musielak Orlicz-Minkowski problem, will be discussed and an existence and uniqueness of solutions to such Minkowski problems will be presented.