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Lifting Trace with Hopf Algebras and Hopf Monads

A Hopf algebra H in a symmetric monoidal category $\mathbb{X}$ has the special ability of lifting many desirable structures and properties of $\mathbb{X}$ to MOD(H), the category of H-modules. Indeed, MOD(H) will be a symmetric monoidal category, and if $\mathbb{X}$ is closed, or star-autonomous, or even compact closed, then MOD(H) will be as well. The antipode of H plays a crucial role in lifting these structures. In this talk, I will explain how Hopf algebras also have the ability of lifting traces. Traced monoidal categories, introduced by Joyal, Street and Verity, are symmetric monoidal categories equipped with a trace operator, which generalizes the classical notion of the trace of matrices in linear algebra. Traced monoidal categories have many applications in mathematics, quantum foundations, and computer science. If $\mathbb{X}$ is a traced monoidal category, then for a Hopf algebra H, MOD(H) will be a traced symmetric monoidal category. In particular, this means that the trace of an H-module morphism is again an H-module morphism. We will also consider the special cases of compact closed categories (where the trace is given by duals), or when the monoidal product is a product (where the trace is given by fixpoints) or a coproduct (where the trace is given by iteration). We will also discuss how this fact also generalizes to the notion of Hopf monads, in the sense of Bruguières, Lack, and Virelizier.

This is joint work with Masahito Hasegawa, and is based on our paper: arXiv:2208.06529