Special Session in Number Theory in Celebration of the 70th Birthday of Ram Murty
Théorie des nombres : à l'occasion du 70e anniversaire de Ram Murty

(Org: **Kumar Murty** (University of Toronto) and/et **Gary Walsh** (Tutte, Ottawa))

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AMIR AKBARY, University of Lethbridge
Constants for Artin-like problems

For an integer $a(\neq 0, \pm 1)$ and a prime $p \nmid a$, the residual index of a mod p, denoted by $i_a(p)$, is the index of the subgroup $\langle a \rangle$ in the multiplicative group $(\mathbb{Z}/p\mathbb{Z})^\times$. The generalized Artin problem asks for establishing an asymptotic formula

$$\sum_{p \leq x} f(i_a(p)) \sim c_{f,a} \mathrm{li}(x),$$

as $x \to \infty$, for suitable arithmetic function $f(n)$, where $c_{f,a}$ is a constant depending on a and f. In 2012, Adam Felix and Ram Murty proved, under the assumption of GRH, a version of the generalized Artin problem, when $f(n)$ satisfies a certain growth condition. We apply the character sums method of Lenstra, Moree, and Stevenhagen to write the constant in the Felix-Murty theorem, when f is multiplicative, as a product indexed over primes times a correction factor. When $f(n)$ is the divisor function $d(n)$, the so-called Titchmarch divisor problem for Kummer fields, we explicitly compute this constant. This is joint work with Milad Fakhari.

MICHAEL BENNETT, University of British Columbia
Powerful numbers in arithmetic progression

I will discuss some recent work on various old problems of Erdos on powerful numbers in arithmetic progression. This is joint work with Prajeet Bajpai and Tsz Ho Chan.

ABHISHEK BHARADWAJ, Queen's University
Linear Relations among special values of L functions

In this talk, we discuss some relations among the values of the digamma function. We also characterise periodic functions f whose special values $L(1, f)$ belong to a restricted vector space. A common theme in both cases is that these values are connected to linear forms in logarithms of numbers in a cyclotomic field. This is a joint work with Ram Murty.

FELIX BARIL BOUDREAU, University of Lethbridge
Arithmetic Rank Bounds for Abelian Varieties

In his career, Ram Murty worked on the problem of bounding the rank of Abelian varieties over number fields, for example, in his 1995 paper *On the rank of* $J_0(N)(\mathbb{Q})$.

In this talk, we examine the analogous problem over function fields. Let K be a function field with perfect constant field k of arbitrary characteristic $p \geq 0$. We give upper bounds, depending on K, on the rank of the Mordell-Weil group over K of any Abelian variety which has trivial K/k-trace. Our result generalizes in various ways a previous theorem by Jean Gillibert (Université de Toulouse) and Aaron Levin (Michigan State University) on elliptic curves over functions fields of characteristic p different from 2 and 3 and is moreover stated under weaker assumptions. We also explore some consequences of our result. This is a joint work with Jean Gillibert and Aaron Levin.

HENRI DARMON, McGill
Green's functions for RM points.

Let τ_1 and τ_2 be real quadratic elements of discriminants D_1 and D_2 $(D_1 \neq D_2)$ of the Drinfeld p-adic upper half plane, let τ_j' be the conjugate of τ_j, let $\Gamma_j \subset \mathbf{SL_2}(\mathbb{Z})$ be the stabiliser of τ_j, and let $(\tau_1, \tau_1') \cdot (\tau_2, \tau_2') \in \{-1, 0, 1\}$ be the topological intersection on the Poincare upper half plane of the two hyperbolic geodesics with these endpoints. Let M_n be the set of 2×2 matrices with integer entries with determinant p. Then the quantity

$$G_n(\tau_1, \tau_2) = \left(\prod_{\gamma \in \Gamma_1 \backslash M_n / \Gamma_2} g(\tau_1, \gamma\tau_2)^{(\tau_1, \tau_1') \cdot \gamma(\tau_2, \tau_2')} \right)^{12}, \quad \text{where} \quad g(z_1, z_2) := \frac{(z_1 - z_2)(z_1' - z_2')}{(z_1 - z_1')(z_2 - z_2')}$$

converges to an element of $\mathbb{Q}_p$ as n tends to infinity. When $p = 2, 3, 5, 7,$ or 13, this quantity is expected to be algebraic and to belong to the compositum of the narrow Hilbert class fields of $\mathbb{Q}(\sqrt{D_1})$ and $\mathbb{Q}(\sqrt{D_2})$. For other p it is expected to be transcendental in general. I will describe a conceptual framework for understanding these assertion by interpreting $G(\tau_1, \tau_2) := \lim_{n \to \infty} G_n(\tau_1, \tau_2)$ as the value of an (exponential) Green's function at the pair (τ_1, τ_2) of RM points.

This is a report on a piece of an ongoing joint project with Jan Vonk.

KARL DILCHER, Dalhousie University
On a result of Koecher concerning Markov-Apéry type formulas for the Riemann zeta function

In 1980 Koecher derived a method for obtaining identities for the Riemann zeta function at odd positive integers, including a classical result for $\zeta(3)$ due to Markov and rediscovered by Apéry. We extend Koecher's method to a very general setting and prove two specific but still rather general results. As applications we obtain infinite classes of identities for alternating Euler sums, further Markov-Apéry type identities, and identities for even powers of π. (Joint work with Christophe Vignat).

HESTER GRAVES, IDA/CCS
The minimal Euclidean function on $\mathbb{Z}[i]$

Ram Murty and his school changed the study of Euclidean algorithms in number fields with class number one by finding growth results on sizes of pre-images of functions. Every Euclidean domain R has a minimal Euclidean function ϕ_R. We introduce the first computable minimal Euclidean function for a non-trivial number field, $\phi_{\mathbb{Z}[i]}$.

HERSHY KISILEVSKY, Concordia University
Non-Zero Central Values of Dirichlet Twists of Elliptic L-Functions

Abstract: We consider heuristic predictions for ''small'' non-zero algebraic central values of twists of the L-function of an elliptic curve E/Q by Dirichlet characters. We provide computational evidence for these predictions and some consequences for an analogue of the Brauer-Siegel Theorem in this context.

MATILDE LALIN, Université de Montréal
The distribution of values of cubic L-functions at $s = 1$

We investigate the distribution of values of cubic Dirichlet L-functions at $s = 1$. Following ideas of Granville and Soundararajan, and Dahl and Lamzouri for quadratic L-functions, we model values of $L(1, \chi)$ with the distribution of random Euler products

$L(1, \mathbb{X})$ for certain family of random variables $\mathbb{X}(p)$ attached to each prime. We obtain a description of the proportion of $|L(1, \chi)|$ that are larger or that are smaller than a given bound, and yield more light into the Littlewood bounds. Unlike the quadratic case, there is a clear asymmetry between lower and upper bounds for the cubic case.

This is joint work with Pranendu Darbar, Chantal David, and Allysa Lumley.

YU-RU LIU, University of Waterloo
Equidistribution of Polynomial Sequences in Function Fields

We prove a function field analog of Weyl's equidistribution theorem of polynomial sequences. Our result covers the case when the degree of the polynomial is greater than or equal to the characteristic of the field, which is a natural barrier when applying the Weyl differencing process to function fields. This is joint work with Thài Hoàng Lê and Trevor D. Wooley.

STEVEN MILLER, Williams College
Combinatorics in Analyzing L-Function Coefficients and Applications to Low-Lying Zeros

Questions on the distribution of coefficients of L-functions can often be reduced to combinatorial questions, where it is not always clear what is the right object to study. I will discuss some earlier joint results with Ram Murty on effective equidistribution of coefficients in elliptic curve families, and discuss how the perspective gained there helped in attacking other problems. These range from extending results for cuspidal newforms to square-free levels and increasing the support of the higher level densities, which lead to the best bounds on vanishing to high order at the central point.

KUMAR MURTY, Fields Institute and University of Toronto
Prime divisors of Fourier coefficients of modular forms

We discuss some old and new results on prime divisors of Fourier coefficients of normalized Hecke eigenforms. Some of the results to be discussed are joint work with Ram Murty. Other collaborators include A.Chow, S. Gun, N. Laptyeva, S. Pujahari and N. Saradha.

RAM MURTY, Queen's University
The large sieve revisited

The large sieve inequality can be viewed as an inequality involving characters of the additive profinite (Prufer) group $\widehat{\mathbb{Z}}$. We will derive a general inequality for arbitrary profinite groups from which the classical large sieve can be deduced as a special case.

BRAD RODGERS, Queen's University
Distances between zeros of L-functions at small and large scales

In this talk I will review some of what is known about the statistical distribution of distances between zeros of the Riemann zeta-function, both at the smallest scale at which such considerations are sensible and at a substantially larger scale. I hope to also offer some speculations about connections to 'large scale' limit theorems in random matrix theory and discuss a connection to work of R. Murty and A. Zaharescu. If there is sufficient time I will also discuss more recent work with J. Lagarias regarding what this information can say about the smallest gaps between zeros.

ABDELLAH SEBBAR, University of Ottawa
Modular Differential Equations

We investigate the modular differential equation $y'' + sE_4y = 0$ on the complex upper half-plane, where E_4 is the weight 4 Eisenstein series and s is a complex parameter. This is equivalent to studying the Schwarz differential equation $\{h, \tau\} = 2sE_4$, where the unknown h is a meromorphic function. On the other hand, such a solution h must satisfy $h(\gamma\tau) = \varrho(\gamma)h(\tau)$, for all $\gamma \in \mathsf{SL}_2(\mathbb{Z})$, where ϱ is a $2-$dimensional complex representation of the modular group and the action on both sides is by linear fractional transformations. Moreover, in order for h to be meromorphic or to have logarithmic singularities at the cusps, it is necessary to have $s = \pi^2 r^2$ with r being a rational number. We show that the nature of the solutions depend on whether ϱ is irreducible or not and on whether its image is finite or not. We will present various techniques to solve the above differential equations in their full generality.

FREYDOON SHAHIDI, Purdue University
Local Langlands Correspondence and the Internal Structure of Arthur Packets

This is a semi-expository talk in which we discuss the local Langlands correspondence (LLC), progress made on it, its connection with the tempered L-packets conjecture to the effect that every such packet has a generic element, and its enhancement in terms of Arthur packets. We also elaborate on certain global consequences and conclude by discussing Jiang's conjecture which is a generalization of the tempered L-packet conjecture to non-tempered Arthur packets and explain some results on it, jointly obtained with Baiying Liu.

CAMERON STEWART, University of Waterloo
On prime factors of terms of binary recurrence sequences

We shall discuss estimates from below for the greatest prime factor of the n-th term of an integer valued non-degenerate binary recurrence sequence.

GARY WALSH, University of Ottawa
Curves with high rank using Pell equations and Murty sums

Generalizing a result of Brown and Meyers, we will describe a fairly large family of curves whose rank is at least two, along with a subfamily, determined by the solvability of certain Pell equations, with rank at least three, and show how Ram Murty Sums can be used to find many curves in this family of rank eight, and some of even higher rank.