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Modular Differential Equations

We investigate the modular differential equation $y'' + sE_4 y = 0$ on the complex upper half-plane, where E_4 is the weight 4 Eisenstein series and s is a complex parameter. This is equivalent to studying the Schwarz differential equation $\{h, \tau\} = 2sE_4$, where the unknown h is a meromorphic function. On the other hand, such a solution h must satisfy $h(\gamma\tau) = \varrho(\gamma)h(\tau)$, for all $\gamma \in \mathrm{SL}_2(\mathbb{Z})$, where ϱ is a $2-$dimensional complex representation of the modular group and the action on both sides is by linear fractional transformations. Moreover, in order for h to be meromorphic or to have logarithmic singularities at the cusps, it is necessary to have $s = \pi^2 r^2$ with r being a rational number. We show that the nature of the solutions depend on whether ϱ is irreducible or not and on whether its image is finite or not. We will present various techniques to solve the above differential equations in their full generality.