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Homological unitality of smooth groupoid algebras

For any Lie group G, the smooth convolution algebra $C_c^\infty(G)$ is nonunital (unless G is discrete), but a celebrated result of Dixmier-Malliavin says the following weaker property holds:

$$\text{Every } \varphi \in C_c^\infty(G) \text{ is a finite sum } \sum f_i * g_i \text{ where } f_i, g_i \in C_c^\infty(G).$$

In a recent article, I extended this result to the case where G is a Lie groupoid. Writing $A = C_c^\infty(G)$, this says exactly that the map $A \otimes A \to A$ defined by convolution product is surjective. Continuing this work, I show that A is homologically unital in the sense of Wodzicki, meaning the bar complex

$$\cdots \longrightarrow A^{\otimes 4} \longrightarrow A^{\otimes 3} \longrightarrow A^{\otimes 2} \longrightarrow A \longrightarrow 0$$

is exact. Wodzikci showed homological unitality is precisely the property needed by an ideal to perform excision in cyclic/Hochschild homology, i.e. the condition for a short exact sequence of algebras to induce a long exact sequence.

For a Lie groupoid G with base X, the concept of an invariant submanifold $Y \subseteq X$ is meaningful (this is consistent with the usual meaning in the group action case). In terms of the smooth convolution algebra, invariant submanifolds manifest as ideals $I_Y^k \subseteq C_c^\infty(G)$, where k encodes an order of vanishing along Y. I furthermore show that I_Y^k is homologically unital for $k = \infty$, which means excision holds for infinite-order vanishing ideals associated to invariant submanifolds. This result gives an organizing principle for calculating cyclic/Hochschild homology: localize the calculation around invariant submanifolds.