
Interaction of discrete and convex geometry with analysis and combinatorics
Interaction de la géométrie discrète et convexe avec l'analyse et la combinatoire
(Org: **Karoly Bezdek** (University of Calgary) and/et **Ferenc Fodor** (University of Szeged, Hungary))

ANDRII ARMAN, University of Manitoba

Upper bounds on the chromatic number of low dimensional spaces

Let $\chi(\mathbb{E}^n)$ denote the chromatic number of the Euclidean space \mathbb{E}^n , i.e., the smallest number of colors needed to color points of \mathbb{E}^n so that no two points unit distance apart are of the same color.

In this talk I will present explicit constructions of colorings of \mathbb{E}^n based on sublattice coloring schemes and establish some new upper bounds. For example, I will provide the construction for the following bounds: $\chi(\mathbb{E}^5) \leq 140$, $\chi(\mathbb{E}^n) \leq 7^{n/2}$ for $n \in \{6, 8, 24\}$, and $\chi(\mathbb{E}^n) \leq 3^n$ for all $n \leq 38$ and $n = 48, 49$.

This talk is based on a joint work with Andriy Bondarenko, Andriy Prymak, and Danylo Radchenko.

KAROLY BEZDEK, University of Calgary

On totally separable packings

A packing by translates of a convex body in Euclidean d-space is called a totally separable packing in short, a TS-packing if any two members of the packing can be separated by a hyperplane which is disjoint from the interior of every member of the packing. TS-packings form a fundamental subfamily of the translative packings in Euclidean d-space. So, it is natural to ask: What do we know about them? The talk surveys the relevant (recent) results.

CHRISTIAN BINGANE, Polytechnique Montreal

Maximal perimeter of a convex small polygon

A small polygon is a polygon of unit diameter. Finding the maximal perimeter of a convex small polygon with a given number of sides is an open problem when the number of sides is a power of two. This presentation discusses recent advances in this problem.

TED BISZTRICZKY, University of Calgary

A COMBINATORIAL CONSTRUCTION OF BI-CYCLIC 4-POLYTOPES

Let $B(p,q,n)$ denote the convex hull of n evenly spaced points on the generalized trigonometric moment curve $(\cos(pt), \sin(pt), \cos(qt), \sin(qt))$, ranging between 0 and 2π , and p and q relatively prime positive integers. If $n=pq$ and $p=q-3$ then there are purely geometrical conditions that yield the face lattice of $B(p,q,n)$.

MANUEL FERNANDEZ, Georgia Institute of Technology

On the ℓ_0 Isoperimetry of Measurable Sets

The Coordinate-Hit-and-run (CHAR) walk is a type of random walk over a measurable set, where at each step a random coordinate of the current point is re-sampled. In a work of Vempala and Laddha, the authors gave the first polynomial bound on the mixing rate of the CHAR walk over convex bodies. As part of their proof strategy, the authors introduced the notion of the ℓ_0 isoperimetric coefficient of a measurable set and provided a lower bound for the quantity in the case of axis-aligned cubes. In this talk we will present some new results regarding the ℓ_0 isoperimetric coefficient of measurable sets. In particular we pin down the exact order of magnitude of the ℓ_0 isoperimetric coefficient of axis-aligned cubes and present a general upperbound of the ℓ_0 isoperimetric coefficient for any measurable set.

FERENC FODOR, University of Szeged, Hungary
Asymptotic expansions for generalized random polygons

There has been quite a lot of work done recently in various generalized models of random polytopes in convex bodies. One such model is when one takes n independent identically distributed uniform random points from a suitable convex body and considers the intersection of all congruent closed balls that contain the points. The resulting intersection is called a random ball-polytope (disc-polygon in the plane). In this talk we discuss the behavior of the vertex number of random disc-polygons. We prove series expansions for the expectation of the vertex number and area of random disc-polygons depending on the degree of smoothness of the boundary of the convex disc. Joint work with N. Montenegro (University of Szeged, Hungary).

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BALAZS GRUNFELDER, University of Szeged, Hungary
On asymptotic properties of generalized random polygons

Let L and K be convex discs. We say that K is L -convex if it is the intersection of all translates of L that contain K . We consider the following probability model: Assume that K is L -convex, and take n independent random points from K according to the uniform probability distribution. The intersection of all translates of L containing the points is a random L -polygon in K . In this talk, we present asymptotic bounds for the variance of the number of vertices and area of such random L -polygons under various geometric conditions on K and L . Joint work with Ferenc Fodor (University of Szeged, Hungary).

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BRETT LEROUX, University of California, Davis
Wendel's theorem and the neighborliness of random polytopes

A convex polytope is k -neighborly if every subset of at most k vertices is a face of the polytope. A well-known feature of random polytopes in high dimension is that they often have a surprisingly high degree of neighborliness. For example, it is known that Gaussian random polytopes are cd -neighborly w.h.p. for some constant $1 > c > 0$ when the number of vertices is proportional to the dimension. Furthermore, work of Donoho and Tanner and Vershik and Sporyshev shows that there is a threshold for the neighborliness of Gaussian random polytopes. We show that a similar thing happens when the vertices are i.i.d. according to an arbitrary absolutely continuous probability distribution on \mathbb{R}^d . As a concrete example, our result implies that if for each d in \mathbb{N} we choose an arbitrary absolutely continuous probability distribution μ_d on \mathbb{R}^d and then set P to be the convex hull of an i.i.d. sample of at most $n = 10d/9$ random points from μ_d , the probability that P is $(d/10)$ -neighborly approaches one as $d \rightarrow \infty$. We will also give an example of a family of distributions which show that this result is close to best possible. The proof relies on a generalization of Wendel's theorem due to Wagner and Welzl.

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BARRY MONSON, University of New Brunswick Fredericton
The Grand Antiprism

Discovered by J. H. Conway and M. Guy in 1965, the *grand antiprism* \mathcal{A} is (for some) the only uniform, convex 4-polytope which is non-Wythoffian. It has 100 vertices and is bounded by 300 regular tetrahedra along with 20 pentagonal antiprisms. The Wikipedia article on \mathcal{A} has some beautiful pictures, and remarks that the symmetry group $S(\mathcal{A})$ of order 400 is the 'Ionic diminished Coxeter group $[[10, 2+, 10]]'$. I will explain how this description is ambiguous, perhaps even wrong (not knowing

the intent of the authors of the article, as of April, 2023). We will give a presentation for $S(\mathcal{A})$, which unexpectedly relates to the regular map $\{10, 10 | 2\}$. And what does ‘non-Wythoffian’ mean, anyway?

KINGA NAGY, University of Szeged, Hungary
Best and random approximations with generalized disc-polygons

In this talk, we consider the asymptotic behaviour of the distance between a convex disc K with sufficiently smooth boundary, and its approximating n -gons, as the number of vertices tends to infinity. We consider two constructions: the best approximating inscribed n -gons with respect to several notions of distance; and random inscribed n -gons obtained by taking the convex hull of n i.i.d. random points chosen from the boundary of K . The asymptotic behaviour of the area deviation of K and the n -gon depend in both cases on the same, geometric limit. The best and random approximating n -gons are defined similarly in the circumscribed case.

We generalize the existing results on linear and spindle convexity to the so-called L -convexity. In the case of inscribed L -polygons, we prove similar asymptotic formulas by generalizing the geometric limits. We also introduce a notion of L -convex duality and consider the properties of the dual disc, which results are then used to prove formulas in the circumscribed case.

Joint work with Viktor Vígh (University of Szeged, Hungary).

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ANDRIY PRYMAK, University of Manitoba
Convex bodies of constant width with exponential illumination number

Borsuk’s number $f(n)$ is the smallest integer such that any set of diameter 1 in the n -dimensional Euclidean space can be covered by $f(n)$ sets of smaller diameter. Currently best known asymptotic upper bound $f(n) \leq (\sqrt{3/2} + o(1))^n$ was obtained by Shramm (1988) and by Bourgain and Lindenstrauss (1989) using different approaches. Bourgain and Lindenstrauss estimated the minimal number $g(n)$ of open balls of diameter 1 needed to cover a set of diameter 1 and showed $1.0645^n \leq g(n) \leq (\sqrt{3/2} + o(1))^n$. On the other hand, Schramm used the connection $f(n) \leq h(n)$, where $h(n)$ is the illumination number of n -dimensional convex bodies of constant width, and showed $h(n) \leq (\sqrt{3/2} + o(1))^n$. The best known asymptotic lower bound on $h(n)$ is subexponential and is the same as for $f(n)$, namely $h(n) \geq f(n) \geq c^{\sqrt{n}}$ for large n established by Kahn and Kalai with $c \approx 1.203$ (1993) and by Raigorodskii with $c \approx 1.2255$ (1999). In 2015 Kalai asked if an exponential lower bound on $h(n)$ can be proved.

We show $h(n) \geq (\cos(\pi/14) + o(1))^{-n}$ by constructing the corresponding n -dimensional bodies of constant width, which answers Kalai’s question in the affirmative. The construction is based on a geometric argument combined with a probabilistic lemma establishing the existence of a suitable covering of the unit sphere by equal spherical caps having sufficiently separated centers. The lemma also allows to improve the lower bound of Bourgain and Lindenstrauss to $g(n) \geq (2/\sqrt{3} + o(1))^n \approx 1.1547^n$.

The talk is based on a joint work with Andrii Arman and Andriy Bondarenko.

EGON SCHULTE, Northeastern University
Skeletal Uniform Polyhedra

A skeletal polyhedron in ordinary space is a finite or infinite discrete structure made up of finite or infinite polygons as faces, with two faces on each edge and a circular vertex-figure at each vertex. Finite faces can be planar or skew, and infinite faces can be linear, zigzag, or helical. A skeletal polyhedron is said to be uniform if its faces are (finite or infinite) regular polygons and its geometric symmetry group is transitive on the vertices. The classification of arbitrary uniform skeletal polyhedra is a rather challenging open problem, but partial results are known. The convex uniform polyhedra are precisely the Archimedean

solids and the prisms and anti-prisms. The classification of the finite uniform polyhedra with planar (convex or star-polygon) faces was essentially obtained in a classical paper by Coxeter, Longuet-Higgins and Miller in 1954, although the completeness of the enumeration was only proved years later, independently, by Skilling and Har'El. We explain how variants of Wythoff's construction applied to the regular or chiral skeletal polyhedra in ordinary space can be exploited to produce highly symmetric, vertex-transitive skeletal polyhedra, which in many cases are new uniform polyhedra. We describe the blueprint for the construction and discuss interesting examples including new skeletal snub polyhedra. This is joint work with Abigail Williams and Tomas Skacel.

ALINA STANCU, Concordia University
On convex bodies with sections of prescribed volume

Part of a larger project with several collaborators, we are studying how certain measurements of non-central sections of a convex body determines uniquely the convex body. Under extra hypotheses, we will show some characterizations of convex bodies in the Euclidean n -space whose $(n - 1)$ -dimensional sections tangent to a strictly, smooth, compact convex set contained in their interior have equal volume.

ALEXANDRA SZABO, University of Szeged, Hungary
On the variance of the volume of random polytopes

We prove an asymptotic upper bound on the variance of the weighted volume of random polytopes which are generated by n independent random points selected from a d -dimensional convex body K according to a certain prescribed probability distribution. We only require K to have relatively weak smoothness properties. Using polar duality we convert these results into asymptotic upper bounds on the variance of the mean width of circumscribed random polyhedral sets about K . Joint work with Ferenc Fodor (University of Szeged, Hungary).

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VIKTOR VIGH, University of Szeged
On random spherical disc-polygons

In 2017 Bárány, Hug, Reitzner and Schneider studied random spherical polytopes that are the spherical convex hull of n independent, uniform random points chosen from a half-sphere. They proved that expectation of the number of facets tends to a constant c_d that depends only on the dimension (as $n \rightarrow \infty$). In 2020 Fodor showed that if we choose independent uniform random points from a unit ball, then the expected number of the facets of the generated uniform random ball-polytope also tends to the constants c_d in any dimension. In this talk we connect these two results in the case when $d = 2$, we study random spherical disc-polygons in a spherical cap of appropriate size, and show that expectation of the number of the edges tends to $c_2 = \pi^2/2$. We also extend the result to a more general case, where we choose the random points from a spherical convex disc with C^2 smooth boundary.

This is a joint work with Kinga Nagy.

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DEPING YE, Memorial University of Newfoundland
The dual Minkowski problem for unbounded closed hypersurfaces

The classical Minkowski problem deals with the characterization of the surface area measure for convex bodies. This problem has been extensively studied in literature and has found fundamental applications in many areas, such as analysis, PDEs, etc. This problem has been extended to various settings, such as the dual Minkowski problem.

An important notion, in analysis, probability, differential geometry, algebraic geometry, singularity theory etc, is the unbounded convex hypersurface, which behaves quite different from its compact relative. Hence, understanding the geometric, algebraic, topological properties for unbounded convex hypersurfaces is in great demand. Among those important topics are the Minkowski type problems for unbounded convex hypersurfaces.

In this talk, I will present some recent progress on the Minkowski type problems for unbounded closed hypersurfaces with concentration on the dual Minkowski problem. I will talk about the setting of this problem and the existence of solutions to this problem.

YIMING ZHAO, Syracuse University

The Minkowski problem in Gaussian probability space

The classical Minkowski problem, which asks for the characterization of surface area measure in Euclidean space with Lebesgue measure, largely motivated the development of elliptic PDEs throughout the last century. In this talk, we will discuss the corresponding problem in Gaussian probability space. Lack of homogeneity and translation invariance make this problem fundamentally different from the classical problem. We will discuss existence results (in all dimensions) as well as uniqueness results (in dimension 2). This is based on joint works with Yong Huang, Dongmeng Xi, and with Shibing Chen, Shengnan Hu, Weiru Liu.