Student Research Talks
Session de présentations étudiantes
(Org: **William Verreault** (Laval))

DIBA HEYDARY, Toronto

AVLEEN KAUR, University of Manitoba

How the Friedrichs angle leads to lower bounds on the minimum singular value

Estimating the eigenvalues of a sum of two symmetric matrices, say $P+Q$, in terms of the eigenvalues of P and Q, has a long tradition. To our knowledge, no study has yielded a positive lower bound on the minimum eigenvalue, $\lambda_{\min}(P+Q)$, when $P+Q$ is symmetric positive definite with P and Q singular positive semi-definite. We derive two new lower bounds on $\lambda_{\min}(P+Q)$ in terms of the minimum positive eigenvalues of P and Q. The bounds take into account geometric information by utilizing the Friedrichs angles between certain subspaces. The basic result is when P and Q are two non-zero singular positive semi-definite matrices such that $P+Q$ is non-singular, then $\lambda_{\min}(P+Q) \geq (1-\cos\theta_F)\min\{\lambda_{\min}(P), \lambda_{\min}(Q)\}$, where $\lambda_{\min}$ represents the minimum positive eigenvalue of the matrix, and θ_F is the Friedrichs angle between the range spaces of P and Q. We will discuss the interaction between the range spaces for some pair of small matrices to elucidate the geometric aspect of these bounds. Such estimates lead to new lower bounds on the minimum singular value of full rank 1×2, 2×1, and 2×2 block matrices in terms of the minimum positive singular value of these blocks. Some examples provided in this talk further highlight the simplicity of applying the results in comparison to some existing lower bounds. This is joint work with S. H. Lui (Manitoba).

SINA MOHAMMAD-TAHERI, Concordia University

Lasso-Inspired Variants of Weighted Orthogonal Matching Pursuit with Applications to Sparse High-Dimensional Approximation

Motivated by recent developments in sparse high-dimensional approximation from Monte Carlo sampling, we propose new weighted generalizations of the Orthogonal Matching Pursuit (OMP) algorithm. Greedy algorithms of this type are more computationally efficient than convex optimization-based methods for small values of the target sparsity and offer a promising way to mitigate the curse of dimensionality. In this work, we propose new theoretically-justified greedy selection criteria that are inspired by variants of the LASSO optimization program. A key issue is the robustness of the optimal choice of the tuning parameter with respect to the measurement noise, which is realized by the square-root LASSO program in the context of convex optimization. We investigate how this property is carried over into the context of LASSO-based OMP methods. Conducting numerical experiments in high-dimensional polynomial approximation, we show the efficacy of the proposed algorithms by studying the recovery error as a function of the algorithm iterations. Moreover, we illustrate settings where the optimal choice of the tuning parameter is more robust against the noise.

CONNOR RIDDLESDEN, University of Lethbridge

Combinatorial Approach to ABV-packets for GL_n

There exists a significant conjecture in the local Langlands correspondence that A-packets are ABV-packets. For the case $G = GL_n$, the conjecture reduces to ABV-packets of Arthur type being singletons, which is a specialisation of the wider conjecture known as the Open-Orbit conjecture. In this introductory talk, we will reduce this problem to a combinatorial study using multisegments, since there exists a natural relationship between the combinatorics of multisegments and the structure of ABV-packets. The talk will focus on introducing the Mœglin-Waldspurger algorithm to compute the Zelevinskii involution and the structure for multisegments of Arthur type. Finally, an outline for the proof of the conjecture that ABV-packets of Arthur type are singletons will be presented using an argument based on numerical invariants and endoscopic decompositions.

SONJA RUZIC, Queen's University
Weyl Modules for Current Lie Superalgebras

The notion of a Weyl module for classical affine algebras, a type of infinite dimensional Lie algebra, was introduced in 2001 by Chari and Pressley. These modules are universal, finite dimensional highest weight modules. We expand these ideas to infinite dimensional Lie superalgebras; in particular to Lie superalgebras of the form $\mathfrak{g} \otimes \mathbb{C}[t]$, where $\mathfrak{g}$ is basic classical. We prove that these Weyl modules are universal, finite dimensional, highest weight $\mathfrak{g}$-modules for $\mathfrak{g} = \mathfrak{sl}_2 \otimes \mathbb{C}[t]$, $\mathfrak{g} = \mathfrak{gl}(1|1) \otimes \mathbb{C}[t]$, and $\mathfrak{g} = \mathfrak{osp}(1|2) \otimes \mathbb{C}[t]$. These three particular cases can be used to generalize the result to $\mathfrak{g} \otimes \mathbb{C}[t]$, where $\mathfrak{g}$ is any basic classical Lie superalgebra. This is part of a work in progress which will be part of my PhD thesis at Queen's University.

ESHA SAHA, University of Waterloo
HARFE: Hard-Ridge Random Feature Expansion

We propose a random feature model for approximating high-dimensional sparse additive functions called the hard-ridge random feature expansion method (HARFE). This method utilizes a hard-thresholding pursuit based algorithm applied to the sparse ridge regression (SRR) problem to approximate the coefficients with respect to the random feature matrix. The SRR formulation balances between obtaining sparse models that use fewer terms in their representation and ridge-based smoothing that tend to be robust to noise and outliers. We prove that the HARFE method is guaranteed to converge with a given error bound depending on the noise and the parameters of the sparse ridge regression model. Based on numerical results on synthetic data as well as on real datasets, the HARFE approach obtains lower (or comparable) error than other state-of-the-art algorithms. As an extension of sparse random feature expansion, we propose an approximation method using time delayed embeddings with random feature matrices when the dynamical system is unknown. We test our method on epidemiological based simulated and real data and show that our method outperforms existing models in terms of seven-day prediction accuracy.

BRUNO STAFFA, University of Toronto
On density and equidistribution of stationary geodesic nets

In 1982, Yau conjectured that every Riemannian 3-manifold has an infinite number of closed immersed minimal surfaces. Successive works of Marques, Neves, Irie, Liokumovich and Song led to the solution of the conjecture in 2018 using Almgren-Pitts minmax theory. The latter is a Morse Theory for the area functional in the space of currents, which are non-smooth generalizations of embedded submanifolds. In the talk, we will focus on studying a 1-dimensional version of Yau's conjecture. In dimension 1, Almgren-Pitts theory produces stationary geodesic nets, which are generalizations of closed geodesics whose domain is a graph Γ instead of S^1. We will discuss two main results about a closed manifold M^n, $n \geq 2$. The first one is that for a generic set of Riemannian metrics on M, the union of all stationary geodesic nets is dense in M. The second one is that for $n = 2$ and $n = 3$ the following equidistribution result holds: for a generic set of metrics g on M, there exists a countable collection of connected and embedded stationary geodesic nets $\{\gamma_i\}_{i \in \mathbb{N}}$ such that

$$\lim_{k \to \infty} \frac{\sum_{i=1}^{k} \int_{\gamma_i} f \mathsf{dL}_g}{\sum_{i=1}^{k} \mathsf{L}_g(\gamma_i)} = \frac{1}{\mathsf{Vol}(M,g)} \int_M f \mathsf{dVol}_g$$

for every smooth function $f : M \to \mathbb{R}$. These results were obtained in collaboration with Yevgeny Liokumovich and Xinze Li respectively.

TONATIUH MATOS WIEDERHOLD, University of Toronto
Graphs of constant balancing number

Given any graph G, there's a large enough complete graph such that every finite colouring of its edges produces a monochromatic copy of G. This fact is called Ramsey's theorem. If instead of a monochromatic copy of G we try to find a copy with say half of the edges of each colour, then the conclusion is only true under certain conditions.

Several interesting families of graphs have been classified regarding these properties, which we discuss in the talk. We also exhibit variations, some new results and a few open problems. The topic will be framed in a general mathematical context spanning extremal graph theory, zero-sum combinatorics and topology.