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Affine group schemes and gradings on algebras by abelian groups

Gradings by groups play an important role in the theory of associative and nonassociative algebras, including Lie and Jordan algebras. Of particular importance are the so-called fine gradings (that is, those that do not admit a proper refinement), because any grading on a finite-dimensional algebra can be obtained from them via a group homomorphism, although not in a unique way. If the ground field is algebraically closed and of characteristic 0, then the classification of fine abelian group gradings on an algebra (up to equivalence) is the same as the classification of maximal quasitori in the algebraic group of automorphisms (up to conjugation). Such a classification is now known for all finite-dimensional simple complex associative, Lie and Jordan algebras. To deal with algebras over an arbitrary field, one can use affine group schemes of automorphisms. I will illustrate this method by connecting abelian group gradings on classical central simple Lie algebras and those on central simple associative algebras with involution. For the latter, a graded version of the classical Wedderburn theorem can be used to reduce the study to the so-called graded-division algebras (meaning that all nonzero homogeneous elements are invertible) and sesquilinear forms over graded-division algebras with involution. This talk is based on joint work with A. Elduque and A. Rodrigo-Escudero.