Global Dynamics and Propagation Phenomena of Biological Systems
Dynamique globale et phénomènes de propagation des systèmes biologiques
(Org: **Shuwen Xue** and/et **Xiaoqiang Zhao** (Memorial))

SHIHENG FAN, Memorial University of Newfoundland
Traveling waves and spreading speed for a Lotka-Volterra competition model in a periodic discrete habitat

In this talk, I will report our recent research on the propagation dynamics of a Lotka-Volterra competition model in a periodic discrete habitat. Under appropriate assumptions, we first show that a semi-trivial equilibrium is globally stable for the spatially periodic initial value problem. Then we establish the existence of the rightward spreading speed and its coincidence with the minimal wave speed for the spatially periodic rightward traveling waves. We further obtain some sufficient conditions for the linear determinacy of the rightward spreading speed.

CHUNYI GAI, The University of British Columbia
Resource-mediated competition between two plant species with different rates of water intake

We propose an extension of the well-known Klausmeier model of vegetation to two plant species that consume water at different rates. Rather than competing directly, the plants compete through their intake of water, which is a shared resource between them. In semi-arid regions, the Klausmeier model produces vegetation spot patterns. We are interested in how the competition for water affects the co-existence and stability of patches of different plant species. We consider two plant types: a "thirsty" species and a "frugal" species, that only differ by the amount of water they consume per unit growth, while being identical in other aspects. We find that there is a finite range of precipitation rate for which two species can co-exist. Outside of that range (when the rate is either sufficiently low or high), the frugal species outcompetes the thirsty species. As the precipitation rate is decreased, there is a sequence of stability thresholds such that thirsty plant patches are the first to die off, while the frugal spots remain resilient for longer. The pattern consisting of only frugal spots is the most resilient. The next-most-resilient pattern consists of all-thirsty patches, with the mixed pattern being less resilient than either of the homogeneous patterns. We also examine numerically what happens for very large precipitation rates. We find that for a sufficiently high rate, the frugal plant takes over the entire range, outcompeting the thirsty plant.

MINGDI HUANG, Xidian University and Memorial University
Propagation dynamics for time-periodic and partially degenerate reaction-diffusion systems

This talk is concerned with the propagation dynamics for partially degenerate diffusion systems with monostable and time-periodic nonlinearity. In the cooperative case, we prove the existence of periodic traveling fronts and the exponential stability of continuous periodic traveling fronts. In the non-cooperative case, we establish the existence of the minimal wave speed of periodic traveling waves and its coincidence with the spreading speed. More specifically, when the system is non-degenerate, the existence of periodic traveling waves is proved by using Schauder's fixed point theorem and the regularity of analytic semigroup; while in the partially degenerate case, due to the lack of compactness and standard parabolic estimates, the existence result is obtained by appealing to the asymptotic fixed point theorem with the help of some properties of the Kuratowski measure of noncompactness. It may be the first work to study the periodic traveling waves of partially degenerate reaction-diffusion systems with non-cooperative and time-periodic nonlinearity.

LEYI JIANG, Memorial University of Newfoundland
Propagation dynamics of a class of epidemic models with different diffusion speeds in a strip region

In this talk, I will report our research on a special SIR model in a stripe region, where infective individuals have a different diffusion speed on one side of the stripe and Dirichlet homogeneous boundary conditions are imposed on the other side. By using the comparison principle and constructing the upper and lower solutions, we establish the existence and uniqueness of the nontrivial steady state under some appropriate conditions. Moreover, we obtain the local uniform convergence of solutions of the Cauchy problem and a threshold-type result. We also prove the existence of the asymptotic spreading speed along the horizontal axis.

LAURENCE KETCHEMEN, University of Ottawa
Global dynamics of populations in fragmented landscapes under monostable and bistable growth dynamics

Many biological populations reside in increasingly fragmented landscapes, where habitat quality may change abruptly in space. A reaction-diffusion model for a single species which grows and disperses in a one-dimensional heterogeneous landscape is presented. The landscape is composed of two homogeneous adjacent patches with different diffusivities and net growth functions (monostable and bistable). An interface condition connects population density and flux between the two patches. We first classify all possible positive steady states using a phase plane approach. We continue by analyzing the stability properties of certain simple possible positive steady states. Numerical simulations reveal fold bifurcations. We end by studying how the movement behaviour can affect the overall dynamics of the population.

THEODORE KOLOKOLNIKOV, Dalhousie University
Stochastic and PDE models of clustering in bacterial colonies

I will discuss a couple of models of pattern formation in bacterial colonies.

The first model is a stochastic agent-based model. Agents move partially at random, and partially towards random neighbours. The latter effect results in clusters. Its continuum limit leads to a 4th order PDE that exhibits a "cluster" steady state.

The second model is a reaction-diffusion system with state-dependent diffusion and results in interface-like solutions.

Joint works with Julien Smith-Roberge, David Iron and Paul Chavy-Waddy

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NA LI, Harbin Institute of Technology and Memorial University
Accelerating propagation in a periodic functional differential equation

In this talk, I will report our recent research on the spreading properties of a class of nonlocal evolution equations with time periodic delay. We first establish fundamental solutions of integro-differential equations with periodic time-delay. In the case where the dispersal kernels are exponentially bounded, we then obtained the existence of spreading speed and its variational characterization. We focus on the case where the kinetic dynamics are of monostable type and the dispersal kernels are algebraically decaying. More precisely, we prove the non-existence of traveling wave solutions and show that the level sets of the solutions eventually locate in between two exponential functions of time.

ZHIMIN LI, Memorial University of Newfoundland
Global dynamics of a nonlocal periodic reaction-diffusion model of Chikungunya disease

In this talk, we propose a nonlocal reaction-diffusion model of Chikungunya disease with seasonality (temperature and rainfall), spatial heterogeneous structure, periodic maturation delay, and periodic extrinsic incubation period. We introduce the basic reproduction number R_m for the vector and the basic reproduction ratio R_0 for the disease to describe the global dynamics of the model system. We further conduct a case study for the Chikungunya transmission in Ceará, Brazil. Our numerical simulations are well consistent with the analytic results. The effects of spatial heterogeneity and some control strategies will be also discussed. This talk is based on a joint work with Dr. Xiao-Qiang Zhao.

MING MEI, Champlain College Saint-Lambert
Sharp traveling waves for time-delayed Fisher-KPP type of degenerate diffusion equations

In this talk, we present the existence of critical traveling waves for the time-delayed Fisher-KPP type of degenerate diffusion equations. These waves are continuous sharp waves with sharp corners caused by the degeneracy of diffusion. The monotone reducing mechanism of the critical waves is also proved, that is, the critical wave speed is decreasing in time-delay. A new phase transform approach is introduced to analyze the delicate local and global behaviors of these critical sharp traveling waves.

MICHAEL PUPULIN, University of Guelph
Travelling wave solutions in a model of Fire blight spread

Fire blight is a bacterial disease that affects apple and pear trees. I present a model to describe the host-pathogen-vector dynamics through a system of two semi-linear reaction-diffusion PDEs coupled to three ODEs. By using Schauder's fixed point theorem in combination with the method of upper and lower solutions, I show that there exists a travelling wave solution connecting the disease free equilibrium with an equilibrium in which no susceptible or infectious hosts remain.

SHIGUI RUAN, University of Miami
Asymptotic and Transient Dynamics of SEIR Epidemic Models on Weighted Networks

In modelling specific infectious diseases, such as COVID-19, populations tend to be inhomogeneous and there are nonlocal interactions as the disease spreads spatially via travelling. Therefore, it is very important to investigate the effects of host heterogeneity on the spatial spread of infectious diseases. We study the effect of population mobility on the transmission dynamics of infectious diseases by considering a susceptible-exposed-infectious-recovered (SEIR) epidemic model with graph Laplacian diffusion; that is, on a weighted network. First we establish the existence and uniqueness of solutions to the SEIR model defined on weighed graph. Then by the means of constructing Liapunov functions, we show that the disease-free equilibrium is globally asymptotically stable if the basic reproduction number is less than unity and the endemic equilibrium is globally asymptotically stable if the basic reproduction number is greater than unity. Finally we apply our generalized weighed graph to Watts-Strogatz network and carry out numerical simulations, which demonstrate that degrees of nodes determine the peak numbers of infectious population as well as the time to reach these peaks. It also indicates that the network has an impact on the transient dynamical behavior of the epidemic transmission. The node degrees determine the peak of infected population, where the greater the degree the higher the peak attains.

YURIJ SALMANIW, University of Alberta
Global dynamics of a diffusive competition model with habitat degradation

It is well known that habitat loss is one of the major contributing factors to the decline of biodiversity worldwide. Partial differential equations offer one method to study the effects of habitat loss on competing species in a spatially explicit setting. Often, we identify three primary forms of habitat loss: degradation, destruction, and fragmentation. In this talk, I will introduce a diffusive competition model subject to habitat degradation and discuss the global dynamics for differing parameter regimes. In the model, the domain (habitat) is partitioned into the healthy region (undisturbed habitat) and the degraded region (due to anthropogenic habitat disturbance). Species follow the Lotka-Volterra competition in the healthy region, while each population experience only decay in the degraded region (not necessarily at the same rate). It is shown that for differing parameter regimes, the competitive exclusion of each species is possible, and moreover there always exists a regime admitting coexistence. I will then briefly discuss the next stages of this project with connections to a habitat destruction problem and the question of fragmentation.

XUEFENG SAN, Lanzhou University and Memorial University
Traveling waves for a periodic epidemic model in a patchy environment

In this paper, we study an epidemic model in a periodic patchy environment with bilinear incidence, where all the parameters share the same period N. When the basic reproduction number R0 is greater than one, we prove the existence of the minimum wave speed c* for nontrivial traveling wave solutions. We also show that there is no nontrivial traveling wave solution when R0 is less than or equal to one and the wave speed c is greater than zero, or R0 is greater than one and c is in the interval (0, c*). We find that the heterogeneity of the environment has important impact on the spatial spread of infectious diseases. More specifically, both the heterogeneity of the transmission rates and the removed rates can increase c* and R0.

RYAN THIESSEN, University of Alberta
Travelling Waves of a New Glioma Invasion Model

I will explore travelling waves for a new glioblastoma multiforme model in this talk. In their Nature paper [4], Osswald and collaborators recently presented a detailed study of in-vivo glioma invasion patterns in the healthy brain tissue of living mice. This paper presents evidence that specialized cancer cells build a network much like a healthy brain neuronal network, which can transmit signals such as calcium waves, forming a glioma-wide communication system. Working jointly with Thomas Hillen, Kevin Painter, and Nadia Loy, our goal is to incorporate the new network formation phenomenon into previous Giloma blastoma models. The model is based on the kinetic model framework, where we can quickly introduce new reaction dynamics for the network formation. We can arrive at a coupled reaction-diffusion equation by making some time scale arguments. From this equation, we will show the existence of Traveling waves with a minimal spreading speed. In addition, we can reduce the system to the classical Fisher-KPP, allowing us to compare the effect of the approximations on the wave speed.

ARVIN VAZIRY, Dalhousie University
Modelling of spatial infection spread through heterogeneous population: from lattice to PDE models

We present a simple model for the spread of an infection that incorporates spatial variability in population density. Starting from first principle considerations, we explore how a novel PDE with state-dependent diffusion can be obtained. This model exhibits higher infection rates in the areas of higher population density, a feature that we argue to be consistent with epidemiological observations. The model also exhibits an infection wave whose speed varies with population density. In addition, we demonstrate the possibility that an infection can "jump" (i.e., tunnel) across areas of low population density towards the areas of high population density. We briefly touch upon the data reported for coronavirus spread in the Canadian province of Nova Scotia as a case example with a number of qualitatively similar features as our model. Lastly, we propose a number of generalizations of the model towards future studies.

XIAOYING WANG, Trent University
How spatial heterogeneity affects transient behavior in reaction-diffusion systems for ecological interactions

Most studies of ecological interactions study asymptotic behavior, such as steady states and limit cycles. The transient behavior, i.e., qualitative aspects of solutions as and before they approach their asymptotic state, may differ significantly from asymptotic behavior. Understanding transient dynamics is crucial to predicting ecosystem responses to perturbations on short time scales. Several quantities have been proposed to measure transient dynamics in systems of ordinary differential equations. Here, we generalize these measures to reaction-diffusion systems in a rigorous way and prove various relations between the non-spatial and spatial effects, as well as an upper bound for transients. This extension of existing theory is crucial for studying how spatially heterogeneous perturbations and the movement of biological species involved affect transient behaviors. We illustrate several such effects with numerical simulations.

XINJIAN WANG, Lanzhou University, Memorial University of Newfoundland
Propagation phenomena for a vector-host disease model

In this talk, I will report our recent research on a vector-host disease system, which models the invasion of vectors and hosts to a new habitat. Combining the uniform persistence idea from dynamical systems with the properties of the corresponding entire solutions, we study the asymptotic spreading phenomena in two different cases:(1) fast susceptible vector; (2) slow susceptible vector when the disease spreads. It turns out that in the former case, the susceptible species spreads faster than the infected species, which leads to two separate fronts with different speeds; while in the latter case, the infected species always catch up with the susceptible species, and three species almost spread as a single front. We also present some numerical simulations to illustrate our analytic results.

JUN-CHENG WEI, University of British Columbia
Traveling Waves to Gross-Pitaevskii Equation, Adler-Moser Polynomials and Kadomtsev-Petviashvili Lump

In this talk, we will consider traveling wave solutions to the Gross-Pitaevskii equation

$$icu_{x_1} = \Delta u + u - |u|^2 u$$

in R^2 or R^3, where the speed $c \in (0, \sqrt{2})$. When c is close to 0 we show that the roots of Adler-Moser polynomials govern the locations of vortices or vortex rings, while when c is close to the sound speed $\sqrt{2}$, the Kadomtsev-Petviashvilli-I lump arises naturally in some rescaled limit. We establish an almost one-to-one correspondence between traveling waves of GP and Adler-Moser polynomials and KP-I. Central to the analysis is the nondegeneracy of roots of Adler-Moser polynomials and KP-I lumps.

SHUWEN XUE, Memorial University of Newfoundland
Propagation dynamics of a parabolic-elliptic chemotaxis model in a time-periodic shifting environment

Chemotaxis models are used to describe the evolution of biological species in response to certain chemical substances in their living environments. In this talk, we first introduce chemotaxis models. Then we talk about the spreading properties of species in a time-periodic shifting environment. Finally, we discuss the existence and nonexistence of periodic forced waves.

YUANXI YUE, Memorial University
Bistable wave speed of a Lotka-Volterra system with nonlocal dispersal

In this talk, I will report our recent research on the speed of traveling wave solutions to a nonlocal dispersal Lotka-Volterra competition model with bistable nonlinearity. Firstly, we establish the value range of the bistable wave speed in terms of spreading speeds of monostable traveling waves in the system. Without any extra requirement on the system parameters, we prove that the bistable wave speed is unique by means of the asymptotic behaviors of the waves near the two stable equilibria. Moreover, comparison principles on wave speeds are established as long as we can find an upper or lower solution with specific speed. Practically, to find or control the speed sign of the propagation (moving direction), we utilize the decay rate of the standing wave (or almost standing wave) to construct test functions so that these functions become upper or lower solutions with almost zero speed for the given parameter sets. Naturally, this provides an insight to obtain (or control) the propagation direction by adjusting the parameter values, which is biologically significant in the study of wave propagation phenomena.