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On density and equidistribution of stationary geodesic nets

In 1982, Yau conjectured that every Riemannian 3-manifold has an infinite number of closed immersed minimal surfaces. Successive works of Marques, Neves, Irie, Liokumovich and Song led to the solution of the conjecture in 2018 using Almgren-Pitts minmax theory. The latter is a Morse Theory for the area functional in the space of currents, which are non-smooth generalizations of embedded submanifolds. In the talk, we will focus on studying a 1-dimensional version of Yau's conjecture. In dimension 1, Almgren-Pitts theory produces stationary geodesic nets, which are generalizations of closed geodesics whose domain is a graph Γ instead of S^1. We will discuss two main results about a closed manifold M^n, $n \geq 2$. The first one is that for a generic set of Riemannian metrics on M, the union of all stationary geodesic nets is dense in M. The second one is that for $n = 2$ and $n = 3$ the following equidistribution result holds: for a generic set of metrics g on M, there exists a countable collection of connected and embedded stationary geodesic nets $\{\gamma_i\}_{i \in \mathbb{N}}$ such that

$$\lim_{k \to \infty} \frac{\sum_{i=1}^{k} \int_{\gamma_i} f d\mathsf{L}_g}{\sum_{i=1}^{k} \mathsf{L}_g(\gamma_i)} = \frac{1}{\mathsf{Vol}(M, g)} \int_M f d\mathsf{Vol}_g$$

for every smooth function $f : M \to \mathbb{R}$. These results were obtained in collaboration with Yevgeny Liokumovich and Xinze Li respectively.