Harmonic Analysis and PDEs
Analyse harmonique et EDP
(Org: **Scott Rodney** (Cape Breton) and/et **Eric Sawyer** (McMaster))

MICHEL ALIXIS, McMaster University
A two-weight T1 theorem for general Calderon-Zygmund operators for doubling weights

I will discuss recent joint work with E. Sawyer and I. Uriarte-Tuero on a two-weight T1 theorem for smooth Calderon-Zygmund operators and doubling weights. We will discuss several key elements in the proof.

SHAOHUA CHEN, Cape Breton University
Global solutions for the 1-D compressible Euler equations with time-dependent damping

In this talk we investigate the Cauchy problem for the 1-D compressible Euler equations with time-dependent damping. We prove the existence of global solutions under the assumptions that the derivatives of initial data are suitable small and the initial volume is large without the condition of small perturbations to the constant initial data. Our approach is based on estimates of the derivatives of Riemann invariants along two characteristic curves.

XING FU, Hubei University (Visiting Scholar of MUN)
Uncertainty Principle on the Lorentz Spaces

In this talk, we introduce a new nonlinear uncertainty principle on the Lorentz spaces with its own interest in quantum mechanics and Schrödinger equation.

JOSE LUIS LUNA GARCIA, McMaster University
Coronizations and big pieces in metric spaces

We compare two ways to approximate a set E in a metric measure space X by a family of subsets $\mathcal{A}$ in X; namely coronizations by $\mathcal{A}$ and big pieces of sets in $\mathcal{A}$. Both are motivated by the work of David and Semmes in the context of uniform rectifiability in $\mathbb{R}^n$. Indeed, this work arose as an attempt to understand these notions in the setting of parabolic geometry in $\mathbb{R}^{n+1}$.

We show that, under some structural assumptions on E and sets in $\mathcal{A}$, if E admits a coronization by sets in $\mathcal{A}$ then E has big pieces of big pieces of sets in $\mathcal{A}$.

This is joint work with Simon Bortz, John Hoffman, Steve Hofmann, and Kaj Nyström.

FLETCHER GATES, McMaster University
Stability of Weighted Haar Wavelets

We will show that weighted Haar bases in an arbitrary doubling measure on $\mathbb{R}$ are stable under small perturbations of the underlying dyadic grid. We also show that there exist non-doubling measures which have similar stability results, but that this stability generally depends on the choice of grid.

KENNEDY OBINNA IDU, Fields Institute
Boundedness of β-numbers and $C^{1,\alpha}$-rectifiability of sets in $\mathbb{R}^n$

The β-numbers were introduced by P. Jones on the travelling salesman problem and used to control the Cauchy singular integral on one-dimensional Lipschitz graphs. G. David and S. Semmes extended this to characterizing those m-dimensional subsets

of $\mathbb{R}^n$, $0 < m < n$, on which we have L^2-boundedness of certain singular integral operators. This is a way of quantifying the notion of rectifiability – describing how much of a set can be covered almost everywhere with countably many Lipschitz graphs or C^1-submanifolds. In this talk we show how this quantity encodes higher order regularity data for sets in $\mathbb{R}^n$. In particular, we show that an almost everywhere local boundedness assumption on the β-numbers suffices for $C^{1,\alpha}$-*rectifiability*, $0 < \alpha < 1$, of subsets of $\mathbb{R}^n$. This is in a joint work with Giacomo Del Nin (Warwick).

ALEX IOSEVICH, University of Rochester
Frame theory and finite point configurations

We are going to discuss some recent developments in the applications of analytic, combinatorial, and number theoretic ideas in the study of finite point configurations to the question of the existence of exponential frames.

DAMIR KINZEBULATOV, Université Laval
Heat kernel bounds and form-boundedness: recent progress

We will talk about a key integral characteristics of a singular vector field (drift) – its form-bound – and its role in the regularity theory of parabolic equations. The regularity results include the following heat kernel bounds:

(1) Gaussian lower bound, provided that the vector field b is form-bounded (e.g. in the weak L^d class or in the Campanato-Morrey class) and has divergence $\operatorname{div} b \geq 0$.

In these assumptions, a Gaussian upper bound is in general invalid.

(2) Gaussian upper bound, provided that b is form-bounded and the positive part of its divergence $(\operatorname{div} b)_+$ is in the Kato class. In these assumptions, the Gaussian lower bound is in general invalid.

(3) Gaussian two-sided bound under a more general form-bounded-type condition on b, assuming that $\operatorname{div} b$ is the Kato class.

The proof uses a rather non-standard variant of Nash's method.

The results presented in this talk is an ongoing collaboration with Yu. A. Semenov.

LYUDMILA KOROBENKO, Reed College
Hypoellipticity via sums of squares

Many results on hypoellipticity of second order operators rely on the assumption that the operator can be written as a sum of squares of vector fields (e.g. Hormander's bracket condition, and Christ's hypoellipticity theorem for infinitely degenerate operators). For operators that are not subelliptic and not sums of squares, hypoellipticity have been only proved in some very special cases, for example, when $L = L_1 + g(x)L_2$ and L_1 and L_2 are subelliptic. In this talk I will address the question of hypoellipticity for a general divergence form operator, whose matrix is comparable, but not necessarily equal, to a diagonal matrix of a special form. The idea is to find sharp sufficient conditions which guarantee that a smooth positive matrix can be written as a sum of squares of positive dyads with sufficient degree of smoothness. Interestingly, this question have not been completely resolved even for scalar positive functions.

SIRAN LI, Shanghai Jiao Tong University
Fractional Sobolev isometric immersions

We discuss $\mathbf{C}^1$-regularity and developability of isometric immersions of flat domains into $\mathbb{R}^3$ enjoying a local fractional Sobolev $W^{1+s,2/s}$ regularity for $2/3 \leq s < 1$, generalising known results on Sobolev and Hölder regimes. Ingredients of the proof include analysis of the weak Codazzi equations of isometric immersions and study of $W^{2,2/s}$-planar deformations with symmetric Jacobian derivative and vanishing distributional Jacobian determinant. (Joint with Reza Pakzad and Armin Schikorra.)

SULLIVAN FRANCIS MACDONALD, McMaster University
Bounded Solutions to p-Poisson Equations

In joint work with S. Rodney, we present sufficient conditions for boundedness of solutions to Dirichlet problems for the p-Poisson equation

$$-\mathrm{div}(|\nabla u|^{p-2}\nabla u) = f$$

on a bounded domain $\Omega \subset \mathbb{R}^n$. In particular, we show that if the data function f belongs to an Orlicz space $L^{\Psi}(\Omega)$ for a Young function Ψ satisfying an integral condition, then any weak solution u is essentially bounded in Ω with

$$\sup_{\Omega} |u| \leq C\|f\|_{L^{\Psi}(\Omega)}.$$

Generalizations of this result to degenerate elliptic p-Poisson equations with drift terms are also discussed.

JOHN-OLIVER MACLELLAN, The University of Alabama

Necessary Conditions for Two Weight Weak Type Norm Inequalities for Multilinear Singular Integral Operators

A central problem in harmonic analysis is to characterize the pairs of weights (u, v) so that a Calderón Zygmund operator maps $L^p(v) \to L^p(u)$. In this talk we discuss the analogous problem for multilinear Calderón Zygmund operators. In particular we establish necessary conditions for weak type norm inequalities provided the kernel satisfies a weak non-degeneracy condition. As an application of our techniques, we show that in general a multilinear Calderón Zygmund does not satisfy a two-weight strong endpoint estimate. (Joint with David Cruz-Uribe.)

BRIAN MCDONALD, University of Rochester

The VC-dimension and point configurations over finite fields

We will discuss connections between Erdős type problems in vector spaces over finite fields and the notion of Vapnik-Chervonenkis dimension. In particular, for a sufficiently large subset $E \subseteq \mathbb{F}_q^2$, we study the VC-dimension of $\mathcal{H} := \{(S-x)\cap E : x \in E\}$, where $S := \{(x_1, x_2) \in \mathbb{F}_q^2 : x_1^2 + x_2^2 = t\}$ for a fixed $t \in \mathbb{F}_q$. We generalize this result to other curves besides the circle, and also present a version for planes in $\mathbb{F}_q^3$.

EVAN MILLER, University of British Columbia

On the regularity of the axisymmetric, swirl-free solutions of the Euler equation in four and higher dimensions

In this talk, we will discuss the axisymetric, swirl-free Euler equation in four and higher dimensions. We will show that in four and higher dimensions the axisymetric, swirl-free Euler equation has properties which could allow finite-time singularity formation of a form that is excluded in three dimensions. We will also consider a model equation that is obtained by taking the infinite-dimensional limit of the vorticity equation in this setup. This model exhibits finite-time blowup of a Burgers shock type. The blowup result for the infinite dimensional model equation heavily suggests that smooth solutions of the Euler equation exhibit finite-time blowup in sufficiently high dimensions.

KABE MOEN, University of Alabama

New bump conditions for commutators

We will present some new two weight bump conditions for commutators of Calderon-Zygmund operators with symbol b. The new aspect of our work is that we consider conditions in which the symbol interacts with the weights. This allows us to consider symbols in subclasses of BMO and obtain several new results. This is a joint work with David Cruz-Uribe and Quan Tran at the University of Alabama.

SCOTT RODNEY, Cape Breton University

Limits of Orlicz Norms

Given a subset Ω of $\mathbb{R}^n$ and a 1-parameter family of Young functions $\{\Psi_j\}_j$, we are interested in when it is the case that

$$(1) \quad \lim_{j\to\infty} \|f\|_{L^{\Psi_j}(\Omega)} = \|f\|_{L^\infty(\Omega)} \tag{1}$$

for a suitable $f \in L^1$. In this talk, I present joint work with A. Mailhot where we show the above equality for families of iterated log-bump Young functions of the form

$$\Psi_j(t) = t^p \left(\prod_{k=1}^{n-1} \log^{(k)}(e_k - 1 + t) \right)^p \left(\log^{(n)}(e_n - 1 + t) \right)^j$$

where $1 \leq p < \infty$ is fixed and $j > 0$. More generally, in joint work with S.F. MacDonald, we give a sharp admissibility criterion for the Young functions Ψ_j that ensures (1) holds.

DONGGEUN RYOU, University of Rochester

A variant of the $\Lambda(p)$-set problem in Orlicz spaces

When p>2, let S be a set of integers and consider trigonometric polynomials whose Fourier coefficients are supported on S. For various sets S, the range of p has been studied where L^p norms of trigonometric polynomials are bounded by their L^2 norms. However, in the opposite direction, we can fix p and think of a set S which satisfies the inequality $\|f\|_p \leq C\|f\|_2$ for some constant C. This set S is called a $\Lambda(p)$-set. In this talk, we will introduce $\Lambda(\Phi)$-sets which are defined in terms of Orlicz norms. And we will discuss some results about $\Lambda(\Phi)$-sets which extends known results about $\Lambda(p)$-sets.

YURIJ SALMANIW, University of Alberta

Eigenvalues in domains with holes: a reaction-diffusion equation approach to habitat fragmentation

The impacts of habitat loss, through habitat degradation and destruction, are well documented and have an undeniable influence on the sustainability of ecosystems. What is less clear, however, is the role habitat fragmentation plays and its relative impact on biodiversity. In this talk, I will explore some of the preliminary results obtained relating to changes in the size of a principal eigenvalue in relation to configurations of bad habitat in a bounded domain. First, I will briefly discuss some strong motivation from recent habitat degradation and destruction models introduced in a competition-diffusion setting. In these models, a primary tool used to determine the global dynamics is the study of a relevant eigenvalue problem obtained through a linearization about the trivial steady state. Then, the question of fragmentation is directly, and quite naturally, related to the configuration of good and bad regions of habitat within a single bounded domain: hence the study of an eigenvalue problem in a domain with holes. I will then conclude with some open questions and future directions for this project.

MOHAMMAD SHIRAZI, McGill University

Prescribing the radial limits of solutions to a PDE

Let S^{n-1} be the unit sphere in $\mathbb{R}^n$, and (θ, r) be the polar representation of points in $\mathbb{R}^n$, for $\theta \in S^{n-1}$, and $r \geq 0$. Moreover, let us call L-harmonic functions the solutions of the PDE $Lu = 0$.

Now, let suppose $U = (U', R)$ is a strictly starlike domain in $\mathbb{R}^n, n \geq 2$, and let F' be an F_σ subset of U', which is of first category if $n = 2$, and polar if $n > 2$. Then, we shall introduce a class of partial differential operators L such that for every function φ continuous on U, there is a L-harmonic function h on U such that, for all $\theta \in F'$, we have

$$\big(h - \varphi\big)((\theta, r)) \to 0.$$

as (θ, r) goes to the boundary of U. That is, the radial limits of h can be prescribed by a continuous function.

This is a joint work with Paul. M. Gauthier.

IGNACIO URIARTE-TUERO, University of Toronto
Two weight norm inequalities for singular and fractional integral operators in $\mathbb{R}^n$

I will report on recent progress on the two weight problem for singular and fractional integral operators in $\mathbb{R}^n$, in particular a local Tb theorem in $\mathbb{R}^n$ for general measures with an energy side condition (joint with C. Grigoriadis, M. Paparizos, E. Sawyer and C.-Y. Shen) and a two weight T1 theorem (with no side conditions) for doubling measures (joint with M. Alexis and E. Sawyer). The talk will be self-contained.

WILLIAM VERREAULT, Université Laval
Unwinding of analytic functions in RKHS and Hardy spaces

We present convergence theorems in H^p and in RKHS for the unwinding series expansion of analytic functions, extending results of Coifman, T. Qian et al. on the Blaschke unwinding series, a nonlinear analogue of Fourier series with a wide range of practical applications.

HONG YUE, Georgia College
Geometric maximal operators and BMO on product bases

This is joint work with Galia Dafni and Ryan Gibara. We consider the problem of the boundedness of maximal operators on BMO on shapes in $\mathbb{R}^n$. We prove that for bases of shapes with an engulfing property, the corresponding maximal function is bounded from BMO to BLO, generalizing a known result of Bennett for the basis of cubes. When the basis of shapes does not possess an engulfing property but exhibits a product structure with respect to lower-dimensional shapes coming from bases that do possess an engulfing property, we show that the corresponding maximal function is bounded from BMO to a space we define and call rectangular BLO.