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Boundedness of β-numbers and $C^{1,\alpha}$-rectifiability of sets in $\mathbb{R}^n$

The β-numbers were introduced by P. Jones on the travelling salesman problem and used to control the Cauchy singular integral on one-dimensional Lipschitz graphs. G. David and S. Semmes extended this to characterizing those m-dimensional subsets of $\mathbb{R}^n$, $0 < m < n$, on which we have L^2-boundedness of certain singular integral operators. This is a way of quantifying the notion of rectifiability – describing how much of a set can be covered almost everywhere with countably many Lipschitz graphs or C^1-submanifolds. In this talk we show how this quantity encodes higher order regularity data for sets in $\mathbb{R}^n$. In particular, we show that an almost everywhere local boundedness assumption on the β-numbers suffices for *$C^{1,\alpha}$-rectifiability*, $0 < \alpha < 1$, of subsets of $\mathbb{R}^n$. This is in a joint work with Giacomo Del Nin (Warwick).