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Heat kernel bounds and form-boundedness: recent progress

We will talk about a key integral characteristics of a singular vector field (drift) – its form-bound – and its role in the regularity theory of parabolic equations. The regularity results include the following heat kernel bounds:

(1) Gaussian lower bound, provided that the vector field b is form-bounded (e.g. in the weak L^d class or in the Campanato-Morrey class) and has divergence $\operatorname{div} b \geq 0$.

In these assumptions, a Gaussian upper bound is in general invalid.

(2) Gaussian upper bound, provided that b is form-bounded and the positive part of its divergence $(\operatorname{div} b)_+$ is in the Kato class. In these assumptions, the Gaussian lower bound is in general invalid.

(3) Gaussian two-sided bound under a more general form-bounded-type condition on b, assuming that $\operatorname{div} b$ is the Kato class.

The proof uses a rather non-standard variant of Nash's method.

The results presented in this talk is an ongoing collaboration with Yu. A. Semenov.