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On Tannaka Recognition and Descent for Topoi

We will recall the constructions of Tannaka theory due to Deligne, involving cogebroides, Hopf algebroids, and their representations. We will show how they can be done over a monoidal category V instead of vector spaces. The basic construction is a cogebroide L from a fiber functor, which becomes a Hopf algebroid when some conditions hold, and a lifting of this fiber functor to the category of L-comodules. A "Tannaka V-recognition theorem" gives conditions under which this lifting is an equivalence.

We will then consider Joyal-Tierney's descent theorem and its application to the structure of topoi using the spatial cover. We take categories of relations on the inverse image of the spatial cover and we obtain a "tannakian fiber functor", but for which V is the category SL of sup-lattices. The Hopf algebroid L coming from this tannakian fiber functor is no other than the formal dual of the localic groupoid G constructed by Joyal-Tierney, thus the descent theorem for topoi becomes equivalent to a Tannaka SL-recognition theorem.

The results I will present can be found in [1], I will also introduce the simpler case in which L is a Hopf algebra and G is a localic group from [2]. Time permitting, I will discuss other reasons (independent of topos theory) to develop Tannaka theory over sup-lattices that I recently beacame aware of.

[1] Dubuc, Szyld, Tannaka theory over sup-lattices and descent for topoi, TAC (2016).

[2] Dubuc, Szyld, A Tannakian Context for Galois Theory, Advances in Mathematics (2013).