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Local Fourier analysis and its application to multigrid for elliptic optimal control problems

In this talk, we first give a brief introduction to local Fourier analysis (LFA). Then we describe applications of LFA to multigrid for control problems, whose discrete linear systems have a saddle-point structure. We propose a novel Braess-Sarazin multigrid relaxation scheme for finite element discretizations of distributed control problems, where we use the stiffness matrix obtained from the five-point finite difference method for the Laplacian as a smoother for the linear system with a mass matrix coefficient arising in the saddle-point system. To solve elliptic sparse optimal control problems with control constraints, discretized by a finite difference method, we study and compare two multigrid relaxation schemes with coarsening by a factor of two, three, and four. We derive LFA optimal smoothing factor for a well-known collective Jacobi relaxation (CJR) scheme. This analysis reveals that the optimal relaxation parameters depend on the mesh size and regularization parameters. To improve CJR, we propose and analyze a new mass-based Braess-Sarazin relaxation scheme for the finite difference discretization, and prove to provide smaller smoothing factors than the CJR scheme for some cases. These schemes are successfully extended to control-constrained cases through the semi-smooth Newton method. Numerical examples are presented to validate our theoretical observations.