STUDC Research Session
Session StudC
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MASOOMEH AKBARI, University of Ottawa
Probabilistic Transitive Closure of Fuzzy Cognitive Maps: Algorithm Enhancement

A *fuzzy cognitive map* (FCM) is made up of factors and direct impacts. In graph theory, a bipolar weighted digraph is used to model an FCM; its vertices represent the factors, and the arcs represent the direct impacts. Each direct impact is assigned a weight in $[0, 1]$ as well as a sign (positive or negative). In the model considered in this work, each weight is interpreted as the probability of the impact. A directed walk from factor F to factor F' is interpreted as an indirect impact of F on F'. The *probabilistic transitive closure* (PTC) of an FCM (or bipolar weighted digraph) is a bipolar weighted digraph with the same set of factors, but with arcs corresponding to the indirect impacts in the given FCM and the weight of each arc equal to the probability of the indirect impact.

FCMs can serve as effective tools to study problems in various fields. They can be used to represent structured knowledge in science, engineering, and the social sciences. Transitive closure provides valuable new information for its corresponding FCM. Unfortunately, computing the PTC of an FCM is computationally hard.

In this talk, we describe a new enhancement of existing algorithms for computing PTC. We show how one can use a separating vertex to reduce the input digraph into smaller components, and how to recover the PTC of the original digraph from the PTCs of the smaller components.

This is joint work with my supervisor, Mateja Šajna.

BRANDON CROFTS, Teachers College, Columbia University
Counting Solutions of $a^2 + pbc = 0$ in a Cube

For a prime p, let $s_p(n)$ be the number of integer triples (a, b, c) which satisfy $a^2 + pbc = 0$, where a, b, c are bounded by natural number n, and p is prime. Some sequences of this form have had limited numbers of terms contributed to the OEIS, while others have had no contributions at all. A non-recursive, generalized algorithm was theorized and developed, to produce the first n terms of the sequence relating to the equation $a^2 + pbc = 0$.

NICK HUANG, University of Toronto
The impact of understanding definitions in students' performances

"Linear algebra is so much different from the math that I have learned in high school." This is a sentence that was said by many university students in their first years. Previous studies have shown that understanding concepts, use of suitable notations in communicating ideas and attitude towards mathematics are major factors that contribute to learning linear algebra. In our study, quantitative and qualitative research is conducted on a class of first year students taking linear algebra at the University of Toronto, Scarborough in order to investigate the impact of mastery in reciting definitions, and the ability to identify and come up examples and counter examples for a particular concept in students' ability to communicate in mathematics. It is not surprising that significant correlations have been found between understanding definition and writing a proof in linear algebra. However, the story turns out to be more complicated and there are multiple factors with different levels of impact in the students' performance. Overall, even though our study supports definition focus approaches to linear algebra, it suggests that focusing on multiple representations on concepts can further improve students learning. In this talk, I will share the methodology and findings of our study. This talk is targeted to anyone with an interest in math education, especially first year instructors, and graduate students with TA responsibility can benefit from our findings.

ALEXIS LEROUX-LAPIERRE, McGill

La théorie de la représentation des algèbres à une frontière (The representation theory of the one boundary algebras)

The Temperley-Lieb algebras and their generalization play an important role both in mathematics and in physics. Their representations turn out to be interesting for certain specific values of a so-called deformation parameter q common to all those families. This talk will be concerned with the one boundary algebras (or blob algebras), a finite dimensional quotient of the affine Temperley-Lieb algebras, when q is a root of unity. More specifically, the quiver of these algebras and a characterization using Loewy diagrams of the projective modules will be given. Joint work with Yvan Saint-Aubin.

Les algèbres de Temperley-Lieb et leurs généralisations jouent un rôle important tant en mathématiques qu'en physique. Leurs représentations s'avèrent intéressantes pour certaines valeurs particulières d'un paramètre de déformation q commun à toutes ces familles. Cette présentation s'intéressera aux algèbres à une frontière (ou algèbre blob), un quotient de dimension finie des algèbres de Temperley-Lieb affines, lorsque le paramètre q est une racine de l'unité. Plus précisément, le carquois de ces algèbres et une caractérisation de certains modules projectifs sous forme de diagramme de Loewy seront donnés. Travail en collaboration avec Yvan Saint-Aubin.

Slides in french, presentation in English.

ANKAI LIU, Queen's University

Quantification of Long Transient Dynamics

Courses and textbooks on mathematical modeling usually focus on the stability of equilibria and asymptotic behaviours of trajectories. However, many interesting phenomena that we would like to model, such as the "honeymoon period" of a disease after the onset of mass vaccination programs, are transient dynamics. Honeymoon periods can last for decades and can be important public health considerations. In many fields of science, especially in ecology, there is growing interest in a systematic study of transient dynamics. In this work we attempt to provide a technical definition of "long transient dynamics" such as the honeymoon period and explain how these behaviours arise in systems of ordinary differential equations. We define a transient center, a point in state space that causes long transient behaviours, and derive some of its properties. In the end we define reachable transient centers, which are transient centers that can be reached from initializations that do not need to be near the transient center.

ROGHAYEH MALEKI, University of Regina

FOUR DIMENSIONAL ASSOCIATION SCHEMES HAVE CYCLOTOMIC CHARACTER VALUES

In 1980, Simon P. Norton posed the Cyclotomic Eigenvalue Question (CEQ) which asks whether the entries of the character table of a commutative association scheme always lie in a cyclotomic number field. The adjacency algebras of association schemes are a special type of standard integral table algebras with integral multiplicites (SITAwIMs). Character formulas for complete graphs, strongly regular graphs, and doubly regular tournaments imply the CEQ is true in dimensions 2 and 3.

In this talk we will show that the values of irreducible characters of SITAwIMs of dimension up to 4 lie in cyclotomic number fields. We also give an example of a SITAwIM with noncyclotomic character values of dimension 5. This is joint work with Allen Herman.

RAJA MILAD, Dalhousie university

Harmonic Analysis on Affine groups and Continuous Wavelet Transform

The set of all invertible affine transformations of a two dimensional real vector space forms a locally compact group G2 that is isomorphic to the semi-direct product group formed when GL2(R) acts on R2 in the obvious manner, where GL2(R) denotes the group of 2 by 2 real matrices with nonzero determinant. We give an explicit decomposition of the left regular representation of G2 as a direct sum of infinitely many copies of a single irreducible representation. We also obtain an analog of the continuous wavelet transform associated to the representation we identify.

FATEMEH POURYAHYA, Ottawa

JÉRÉMIE TURCOTTE, McGill University
Bounding the cop number of small graphs

We introduce the game of Cops and Robbers, played on graphs. We present recent progress on graphs which are extremal for the cop number. It is well known that the smallest connected graph for which 3 cops are needed to capture the robber is the Petersen graph. Using both formal and computational methods, we determine the minimum order of connected 4-cop-win graphs, which confirms a conjecture of Andreae (1986), and later of Baird et al. (2014), and work towards the uniqueness of such graphs. Based on joint work with Samuel Yvon.

AXEL TURNQUIST, New Jersey Institute of Technology
Optimal Transport on the Sphere

Recently, much progress has been made using wide-stencil finite-difference schemes to provide convergence frameworks for optimal transport and related Monge-Ampère-type PDE problems. One particular merit of these methods lies in their ability to deal with viscosity solutions, as first hinted at in a 1991 paper by Barles-Souganidis. However, these methods have so far not been adapted to geometries other than subsets of Euclidean space. Recent applications including global moving-mesh methods in meteorology and the reflector antenna problem have inspired work on the optimal transport problems on the sphere, which utilize the squared geodesic cost and a negative logarithmic cost, respectively. Here, we construct a numerical convergence framework for optimal transport on the sphere with both cost functions in mind. We show that a wide-stencil finite-difference scheme can be constructed to solve the optimal transport problem on the sphere that has guaranteed convergence to a modified PDE in both smooth and non-smooth cases, where in the latter case we must construct underestimating schemes. We supplement this theory with a particular construction of the scheme which satisfies the hypotheses of the convergence theorem and demonstrate its effectiveness at computing heretofore unsolved problems on the sphere.