Symplectic geometry
Géométrie symplectique
(Org: **Lisa Jeffrey** (Toronto), **Derek Krepski** (Manitoba) and/et **Luke Volk** (Ottawa))

PETER CROOKS, Northeastern University
Hamiltonian reduction along a pre-Poisson subvariety

Topological quantum field theories (TQFTs) serve to inspire many important constructions in geometry and representation theory. A concrete example of this inspiration comes from a paper of Moore and Tachikawa, where the authors conjecture the existence of a certain TQFT taking values in holomorphic symplectic varieties. Verifying this conjecture amounts to constructing a particular family of holomorphic symplectic varieties indexed by the natural numbers, the so-called *Moore–Tachikawa varieties*. Ginzburg and Kazhdan thereby prove Moore and Tachikawa's conjecture.

I will realize the Ginzburg–Kazhdan construction as an instance of "Hamiltonian reduction along a pre-Poisson subvariety", a procedure developed jointly with Maxence Mayrand. This reduction procedure also encompasses Marsden–Weinstein reduction, symplectic implosion, Mikami–Weinstein reduction, and hyperkähler slices, all of which I will explain if time permits.

This represents joint work with Maxence Mayrand.

MEGUMI HARADA, McMaster University
A local normal form for Hamiltonian Poisson-Lie group actions

We present a local normal form for Hamiltonian actions of Poisson-Lie groups K on a symplectic manifold equipped with a K^*-valued moment map, where K^* is a dual Poisson-Lie group to K. Our proof uses the delinearization theorem of Alexeev, Meinrenken, and Woodward, which relates a classical Hamiltonian action of K with $\mathfrak{k}^*$-valued moment map to a Hamiltonian action with a K^*-valued moment map, via a deformation ("delinearization") of symplectic structures. We obtain our main result by proving a "delinearization commutes with symplectic reduction" theorem which is also of independent interest, and then putting this together with the local normal form theorem for classical Hamiltonian actions wtih $\mathfrak{k}^*$-valued moment maps. A key ingredient for our main result is the delinearization $\mathcal{D}(\omega_{can})$ of the canonical symplectic structure on T^*K. Time permitting, I will briefly describe some steps toward explicit computations of $\mathcal{D}(\omega_{can})$. This talk is based on joint work with an undergraduate, Mr. Aidan Patterson, and Jeremy Lane, for an NSERC summer USRA project.

JACQUES HURTUBUISE, McGill University
Torsors over the moduli of bundles

If M is the moduli space of bundles over a Riemann surface X, then we can define two torsors for T^*M: - the first is the moduli C of pairs (bundles, flat connections); -the second involves taking the determinant line bundle L over M, and considering on L, the bundle $Conn$ of connections (the thing of which a section would be a connection on L). Curiously the two (C and $Conn$) are equivalent as torsors, and even symplectomorphic. The identifications go by choosing a pair of canonical and seemingly unrelated sections over M; we do this in two ways. The identification seems to be fairly robust, as it is independent of which pair is chosen.

A similar picture holds over the bigger space of pairs (curve, bundle on that curve), that is, allowing the curve to move.

(joint work with Indranil Biswas, and Volodya Rubtsov)

PETER KRISTEL, University of Manitoba
The smooth spinor bundle on loop space

Given a smooth manifold, M, there is a hierarchy of interesting extra structures that M may or may not admit: $\mathrm{metric} \leftarrow \mathrm{orientation} \leftarrow \mathrm{spin\ structure} \leftarrow \mathrm{string\ structure} \leftarrow \dots$, these structures correspond to reductions of the structure group of

TM along the Whitehead tower of the orthogonal group $\mathrm{GL}(d) \cong \mathrm{O}(d) \leftarrow \mathrm{SO}(d) \leftarrow \mathrm{Spin}(d) \leftarrow \mathrm{String}(d) \leftarrow \ldots$. Manifolds which admit a spin structure have extremely rich geometry, and are still being studied intensively. Manifolds with a string structure, on the other hand, are not nearly as well understood. One of the main difficulties is that $\mathrm{String}(d)$ is not a Lie group. In the eighties, Killingback argued that a string structure on M induces a spin structure on the smooth loop space $LM = C^\infty(S^1, M)$. Seemingly, this exchanges one difficulty for another, because LM is infinite dimensional, and classical spin geometry does not apply. In this talk I will explain how to adapt one of the fundamental notions of spin geometry, namely the spinor bundle, to this infinite dimensional case.

JEREMY LANE, McMaster University
The cohomology rings of Gelfand-Zeitlin fibers

Gelfand-Zeitlin systems are completely integrable systems on unitary and orthogonal coadjoint orbits that share many features with toric systems. One thing that distinguishes them from toric systems is the presence of moment map fibers which are not tori. As some of the non-toric Gelfand-Zeitlin fibers are Lagrangian, they may play an important role in the geometric quantization and Fukaya category of unitary and orthogonal coadjoint orbits. They are also interesting from the perspective of topology of integrable systems on symplectic manifolds. This motivates a better understanding of the topology of these fibers. In this talk I will present recent work with Jeffrey Carlson in which we computed the cohomology rings of all Gelfand-Zeitlin fibers. Following earlier work by other authors, our results can be phrased nicely in terms of the combinatorics of the associated Gelfand-Zeitlin polytopes.

YIANNIS LOIZIDES, Cornell University
Hamiltonian loop group spaces and a theorem of Teleman and Woodward

I will revisit a theorem of Teleman and Woodward that computes the index of the Atiyah-Bott K-theory classes on the moduli space of G-bundles on a curve. I will describe a perspective on this theorem that is based on Hamiltonian loop group spaces, symplectic geometry, and index theory.

MYKOLA MATVIICHUK, McGill University
Forty families of log symplectic forms on CP^4

I will explain how the local Torelli theorem from Brent Pym's talk describes (not necessarily toric) deformations of toric log symplectic forms on complex projective spaces. I will introduce smoothing diagrams, which are certain graphs with decorations that encode such deformations, discuss combinatorial rules that govern them, and present a complete classification of smoothing diagrams for the case of CP^4. The obtained list of 40 smoothing diagrams amounts to 40 families of log symplectic forms on CP^4, most of which are new. Time permitting, I will discuss how to read off geometric properties of the obtained log symplectic forms from the smoothing diagrams. This is joint work with Brent Pym and Travis Schedler.

ECKHARD MEINRENKEN, University of Toronto
On the Virasoro coadjoint action

The Virasoro algebra $\mathfrak{vir}$ is the non-trivial central extension of the Lie algebra of vector fields on the circle. There is a well-known 1-1 correspondence between the coadjoint orbits in the level 1 subspace $\mathfrak{vir}^*_1 \subset \mathfrak{vir}^*$ and conjugacy classes in a certain open subset $U \subset \widetilde{\mathrm{SL}}(2, R)$. We extend this correspondence by taking into account the geometric structure, giving a Morita equivalence between the Poisson structure on $\mathfrak{vir}^*_1$ and the Cartan-Dirac structure on U. (Joint work with Anton Alekseev.)

BRENT PYM, McGill University
A local Torelli theorem for log symplectic manifolds

A log symplectic manifold is a holomorphic symplectic manifold whose two-form is allowed to have logarithmic poles on a hypersurface. I will describe the structure of the moduli space of such manifolds near the locus of log symplectic manifolds whose divisor has normal crossings. Generically, the moduli space is smooth and parameterized by the periods of the two-form, in parallel with the classical local Torelli theorems for compact hyperkähler manifolds. However, when the periods satisfy certain integer-linear conditions, we find new irreducible components of the moduli space corresponding to structures where the normal crossings divisor is deformed to a more interesting singularity type (e.g. elliptic). This talk is based on joint work with Mykola Matviichuk and Travis Schedler, and is a prequel to Matviichuk's talk, which will explain how these techniques can be used to obtain nontrivial global classification results, using projective spaces as an example.

STEVEN RAYAN, University of Saskatchewan
Integrability and symplectic duality for generalized hyperpolygons

In this talk, I will construct a generalization of hyperpolygon space from a comet-shaped quiver. The resulting Nakajima quiver variety can be interpreted as a distinguished subvariety of a moduli space of meromorphic Higgs bundles on a punctured curve. I will discuss how this space of generalized hyperpolygons inherits, for complete and minimal flags, a Gelfand-Tsetlin-type integrable system from the reduction of a product of cotangent bundles of (partial) flag varieties, as shown in joint work with Laura Schaposnik. Inspired by this work, I will introduce a conjectural Coulomb branch for the space of generalized hyperpolygons, which is one step towards fully realizing symplectic duality in this setting.

REYER SJAMAAR, Cornell University
Toric symplectic stacks

I will outline B. Hoffman's theory of toric symplectic stacks, which are classified by simple, not necessarily rational, convex polytopes equipped with some additional combinatorial data. The orbit space of a toric symplectic stack is a toric symplectic quasifold in the sense of Prato. Hoffman's results extend Delzant's classification of compact toric symplectic manifolds. His theory is distinct from the theory of toric stacks developed by algebraic geometers.