Representations of p-adic groups and Langlands correspondences
Représentations de groupes p-adiques et correspondances de Langlands
(Org: **Karol Koziol** (Michigan) and/et **Monica Nevins** (Ottawa))

JEFF ADLER, American University
Regular Bernstein blocks

Let G be a connected reductive group over a nonarchimedean local field F. The Bernstein decomposition expresses the category of smooth, complex representations of $G(F)$ as a product of full subcategories, called Bernstein blocks, containing representations that all have the same depth. One hopes that, in some generality, a positive-depth Bernstein block for $G(F)$ will be equivalent to a depth-zero Bernstein block for $G^0(F)$, where G^0 is some twisted Levi F-subgroup of G. I will outline some cases where the hope is realized. This is joint work with Manish Mishra.

ERAN ASSAF, Dartmouth College
Existence of Invariant Norms in p-adic Representations of $GL_2(F)$ with Large Weights

Let F be a finite extension of $\mathbb{Q}_p$ and let q be the cardinality of its residue field. The Breuil-Schneider conjecture for $G = GL_n(F)$ predicts a necessary and sufficient condition for the existence of an invariant norm on $\rho \otimes \pi$, where ρ is an irreducible algebraic representation of G and π is an irreducible smooth representation of G over $\overline{F}$. The conjecture is still open, even when $n = 2$, if π is a principal series representation. In this case, assuming π is unramified and $\rho = \mathrm{Sym}^k \otimes \det^m$, it had been verified by Breuil and De Ieso when $k < q$, and these results have been extended to the range $k < q^2/2$, imposing some technical conditions on π and k. In the talk we will provide a new proof of these results, and remove some of the technical conditions.

ADÈLE BOURGEOIS, Carleton University
Supercuspidal L-packets of G_2 in Relation to Those of SO_8 and PSO_8

Little is known about the Local Langlands Correspondence (LLC) for the exceptional group G_2 over a non-archimedean local field. However, G_2 can be realized as a twisted endoscopic group of PSO_8, which in turn is closely related to SO_8. Because the LLC for SO_8 is well-known, the idea is to establish connections between the L-parameters and L-packets of $\mathrm{SO}_8, \mathrm{PSO}_8$ and G_2. In particular, one can start by restricting their attention to supercuspidal L-parameters and L-packets in order to take advantage of Kaletha's recent parameterizations. This talk will focus on our recent progress in establishing a clear relationship between the supercuspidal L-packets of the three groups at play.

ROBERT CASS, Harvard University
Geometrization of mod p Hecke algebras

We will give an overview of three applications of techniques from the geometric Langlands program toward the study of mod p Hecke algebras. The first is a mod p version of the geometric Satake equivalence which provides a geometric version of Herzig's mod p Satake isomorphism. The second geometrizes an isomorphism due to Ollivier and Vignéras describing the center of the Iwahori mod p Hecke algebra. The third is joint work with Cédric Pépin and geometrizes the mod p Satake transform with respect to a general Levi subgroup.

CLIFTON CUNNINGHAM, University of Calgary
Vogan's geometric perspective on local L-packets and A-packets

This talk concerns Vogan's geometric perspective on the Langlands correspondence, which identifies irreducible admissible representations of a p-adic group $G(F)$ and its pure inner forms with simple perverse sheaves on a moduli space of Langlands parameters. We will explain how to identify L-packets using this perspective and then explain Vogan's conjecture on how to identify A-packets using the geometry of this moduli space. We will then present evidence for this conjecture and progress toward a proof. This talk includes examples all these notions using unipotent representations of $\mathrm{SO}_5(F)$ and $\mathrm{G}_2(F)$.

JESSICA FINTZEN, University of Cambridge and Duke University
Representations of p-adic groups

The Langlands program is a far-reaching collection of conjectures that relate different areas of mathematics including number theory and representation theory. A fundamental problem on the representation theory side of the local Langlands program is the construction of all (irreducible, smooth) representations of p-adic groups.

I intend to provide an overview of our understanding of the complex and mod-ℓ representations of p-adic groups and outline recent developments and applications.

STELLA GASTINEAU, Boston College
Diving into the Shallow End

In 2013, Reeder-Yu gave a construction of supercuspidal representations by starting with stable characters coming from the shallowest depth of the Moy-Prasad filtration. In this talk, we will be diving deeper—but not too deep. In doing so, we will construct examples of supercuspidal representations coming from a larger class of "shallow" characters. Using methods similar to Reeder-Yu, we can begin to make predictions about the Langlands parameters for these representations.

TOM HAINES, University of Maryland
Geometry of affine Schubert varieties and applications

Classical Schubert varieties are orbit-closures of a Borel subgroup acting on a partial flag variety attached to a connected reductive group. They play a central role in representation theory and combinatorics. Their geometric properties – whether they are normal, Cohen-Macaulay, or Frobenius-split; when they are singular, and what kind of singularities arise, etc – have been intensively studied and are now well understood. Affine Schubert varieties are similar objects but attached to a loop group rather than a group. They play a role in representation theory, mathematical physics, and in geometric approaches to automorphic forms. In the last 20 years they have been studied in large part because of their connection to certain Shimura varieties through the theory of Rapoport-Zink local models. But some key geometric properties – including normality – remain somewhat mysterious to this day, at least in some positive characteristic settings. This talk will survey some recent advances in the study of affine Schubert varieties, especially the surprising fact that almost all affine Schubert varieties in "bad" positive characteristic are not normal. We will connect this to the Langlands program by explaining how these results are used to understand the geometry of certain Shimura varieties.

SEAN HOWE, University of Utah
p-adic automorphic forms for GL_2

There are (at least) three natural spaces of p-adic automorphic forms for $\mathrm{GL}_2/\mathbb{Q}$: Katz-Serre p-adic modular forms (and their perfected variant), Serre's quaternionic forms, and completed cohomology. Away from p all three have the same completed Hecke algebra, while at p completed cohomology admits an action of $\mathrm{GL}_2(\mathbb{Q}_p)$ and the other two admit actions of closely related p-adic groups. For p-adic modular forms and completed cohomology, the representations of these p-adic groups appearing in a fixed eigensystem are well-understood (by the q-expansion principle and Emerton's local-global compatibility, respectively), while the structure in the quaternionic case remains more mysterious. In this talk, we explain how Pan's recent results on the ubiquity of overconvergent modular forms can be used to extract some information about this structure.

PETER LATHAM, University of Ottawa
The inertial Langlands correspondence

The inertial Langlands correspondence is a modification of the local Langlands correspondence which relates Bushnell–Kutzko types to representations of the inertia group. I will explain a refinement of this correspondence which includes the monodromy action of Langlands parameters. This is proved by establishing a precise connection between typical representations and the decomposition of parabolically induced representations.

DANIEL LE, Purdue University
A mod p local-global compatibility result for generic Fontaine-Laffaille representations

By work of Khare-Wintenberger, Colmez, Emerton, and others, the commuting actions of $\mathrm{GL}_2(\mathbb{A})$ and $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the $\overline{\mathbf{F}}_p$-cohomology of the tower of modular curves realizes a mod p Langlands correspondence, characterized by the Eichler-Shimura relation at good primes and Colmez's Montreal functor at p. With no conjectural formulation of a mod p Langlands correspondence for $\mathrm{GL}_n(\mathbb{Q}_p)$ at present, it is natural to ask if a local $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$-representation can be recovered from the corresponding $\mathrm{GL}_n(\mathbb{Q}_p)$-representation appearing in the cohomology of an appropriate adelic quotient. We give an affirmative answer in some generic Fontaine-Laffaille cases (also allowing unramified extensions of $\mathbb{Q}_p$). This is joint work with Le Hung, Morra, Park, and Qian.

GIL MOSS, University of Utah
Toward a local Langlands correspondence in families

The local Langlands correspondence connects representation of p-adic groups to Langlands parameters, which are certain representations of Galois groups of local fields. In recent work with Dat, Helm, and Kurinczuk, we have shown that Langlands parameters, when viewed through the right lens, occur naturally within a moduli space over Z[1/p], and we can say some things about the geometry of this moduli space. This geometry should be reflected in the representation theory of p-adic groups, on the other side of the local Langlands correspondence. The "local Langlands in families" conjecture describes the moduli space of Langlands parameters in terms of the center of the category of representations of the p-adic group– it was proved for GL(n) in 2018. The goal of the talk is to give an overview of this picture, including current work in-progress, with some discussion of the relation to recent work of Zhu and Fargues-Scholze.

RACHEL OLLIVIER, UBC
The pro-p-Iwahori Hecke Ext-algebra of $SL(2, \mathbb{Q}_p)$

Given a p-adic reductive group G and its (pro-p) Iwahori-Hecke algebra H, we are interested in the link between the category of smooth representations of G and the category of H-modules. When the field of coefficients has characteristic zero this link is well understood by work of Bernstein and Borel.

In characteristic p things are still poorly understood and the role of the pro-p Iwahori-Hecke algebra H is played by a differential graded Hecke algebra. In particular, by work of Peter Schneider, the module category over the d.g. Hecke algebra is equivalent to the derived category of smooth representations of G. Unlike in the case of H, we know little about the structure of this d.g. Hecke algebra.

In this talk I will report on joint work with Peter Schneider where we study the cohomology of the d.g. Hecke algebra. When $G = SL(2, \mathbb{Q}_p)$ we now understand its structure well enough to deduce some properties of mod p representations of $SL(2, \mathbb{Q}_p)$. We also have results for certain more general groups.