Recent Developments in Gröbner Geometry
Développements récents en géométrie de Gröbner
(Org: **Megumi Harada**, **Jenna Rajchgot** and/et **Sergio Da Silva** (McMaster))

LAURA ESCOBAR, Washington University in St. Louis
Gröbner bases for a family of symmetric determinantal ideals

I will discuss a class of combinatorially-defined polynomial ideals which are generated by minors of a generic symmetric matrix. Each ideal in the class encodes the coordinates and equations for neighborhoods of certain type C Schubert varieties at torus fixed points. Our main result gives Gröbner bases for these ideals. The first part of the talk will focus on motivation and connections to both the Schubert variety literature and the commutative algebra literature. Then I will discuss our Gröbner basis result as well as combinatorial formulas for their multigraded Hilbert series in terms of pipe dreams.

This is joint work with Alex Fink, Jenna Rajchgot, and Alexander Woo.

ZACH HAMAKER, University of Florida
Grobner degeneration for skew-symmetric matrices

Since the introduction of Grobner geometry in Knutson and Miller's breakthrough work on matrix Schubert varieties, the technique has been employed to study many related spaces. In a major advance building on previous work joint with the speaker, Marberg and Pawlowski used these methods to describe K-theory representatives for skew-symmetric matrix Schubert varieties. In this talk, we explore further work in this setting. This includes joint work with Anna Weigandt.

PATRICIA KLEIN, University of Minnesota
A proof of a conjecture about Schubert determinantal ideals

Knutson and Miller (2005) showed that the Fulton generators form Gröbner bases of Schubert determinantal ideals under any anti-diagonal term order. Gröbner bases of diagonal term orders have proved much more elusive. Recently, Hamaker, Pechenik, and Weigandt conjectured that a generating set they named the CDG generators form a diagonal Gröbner basis if and only if 8 permutation patterns are avoided. In this talk, we will use the relationship between Gorenstein liaison and geometric vertex decomposition, explored the speaker's previous work with Rajchgot, to gain intuition for why these 8 patterns must be avoided and to sketch a proof of the conjecture.

ALLEN KNUTSON, Cornell University
Partial ordinary, and bumpless, pipe dreams

First I'll define "partial pipe dreams", which is somewhere between a permutation and a pipe dream for that permutation. To each such D I'll associate a variety $Y_D \subseteq Mat_n$ that is correspondingly between a matrix Schubert variety and a coordinate subspace. Then the inductive theorem is that if we revlex the matrix variable at an "outer corner" (i, j) of D, Y_D degenerates to a union of various $Y_{D'}$ where the pipe dream part of D' is that of D plus one more tile at (i, j). Then I'll talk about the projective dual statement, lexing partial bumpless pipe dreams. Time permitting, I'll talk about joint work in progress with P. Zinn-Justin interpolating between the ordinary and bumpless pictures.

EMMANUEL NEYE, University of Saskatchewan
Gröbner bases for Kazhdan-Lusztig ideals

Schubert determinantal ideals are a class of generalized determinantal ideals which include the classical determinantal ideals. In this talk, we use the approach of "Gröbner basis via linkage" to give a new proof of a well-known result of Knutson and

Miller: the essential minors of every Schubert determinantal ideal form a Gröbner basis with respect to a certain term order. We also adapt the Gröbner basis via linkage technique to the multigraded setting and use this to show that the essential minors of every Kazhdan-Lusztig ideal form a Gröbner basis with respect to a certain term order, thereby giving a new proof of a result of Woo and Yong.

OLIVER PECHENIK, University of Waterloo
Gröbner Geometry of Schubert Polynomials Through Ice, Part I

The geometric naturality of Schubert polynomials and the related combinatorics of pipe dreams was established by Knutson and Miller (2005) via antidiagonal Gröbner degeneration of matrix Schubert varieties. We consider instead diagonal Gröbner degenerations. In this dual setting, Knutson, Miller, and Yong (2009) obtained alternative combinatorics for the class of vexillary matrix Schubert varieties. We will discuss general diagonal degenerations, relating them to an older formula of Lascoux (2002) in terms of the 6-vertex ice model. Lascoux's formula was recently rediscovered by Lam, Lee, and Shimozono (2018), as "bumpless pipe dreams." We will explain this connection and discuss conjectures and progress towards understanding diagonal Gröbner degenerations of matrix Schubert varieties.

COLLEEN ROBICHAUX, University of Illinois at Urbana-Champaign
Castelnuovo-Mumford regularity and Kazhdan-Lusztig varieties

We give an explicit formula for the degree of a vexillary Grothendieck polynomial. We apply our work to compute the Castelnuovo-Mumford regularity of certain matrix Schubert varieties. We also derive a formula for the regularity of mixed one-sided ladder determinantal ideals. This is joint work with Jenna Rajchgot and Anna Weigandt.

INFORMAL SOCIALIZATION,

ANNA WEIGANDT, University of Michigan
Gröbner Geometry of Schubert Polynomials Through Ice, Part II

The geometric naturality of Schubert polynomials and the related combinatorics of pipe dreams was established by Knutson and Miller (2005) via antidiagonal Gröbner degeneration of matrix Schubert varieties. We consider instead diagonal Gröbner degenerations. In this dual setting, Knutson, Miller, and Yong (2009) obtained alternative combinatorics for the class of vexillary matrix Schubert varieties. We will discuss general diagonal degenerations, relating them to an older formula of Lascoux (2002) in terms of the 6-vertex ice model. Lascoux's formula was recently rediscovered by Lam, Lee, and Shimozono (2018), as "bumpless pipe dreams." We will explain this connection and discuss conjectures and progress towards understanding diagonal Gröbner degenerations of matrix Schubert varieties.

ALEXANDER WOO, University of Idaho
Delta-Springer fibers

We introduce a family of compact varieties $Y_{n,\lambda,s}$ that generalize the Springer fibers in type A. We show that they have a paving by affines and use properties of this paving to give a presentation for their cohomology rings. These cohomology rings have an action of S_n with the top dimensional cohomology being an induced Specht module. In the case where $\lambda = (1^k)$ and $s = k$, the cohomology ring is the ring constructed by Haglund-Rhoades-Shimozono whose graded Frobenius characteristic is the symmetric function $\omega(\Delta'_{e_{k-1}} e_n(q, 0))$.

This is joint work with Sean Griffin (ICERM/UC Davis) and Jake Levinson (SFU).

ALEX YONG, University of Illinois at Urbana-Champaign
Hilbert-Samuel multiplicities of Schubert varieties

I'll revisit some older (still unsolved) conjectures with Li Li (Oakland University) and Alexander Woo (University of Idaho) on Hilbert-Samuel multiplicities of Schubert varieties. This talk concerns Grobner and tangent cone degenerations of Kazhdan-Lusztig ideals/varieties.