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Maximizing the sum of angles between pairs of lines in Euclidean space

Choose N unoriented lines through the origin of $\mathbf{R}^{d+1}$. Suppose each pair of lines repel each other with a force whose strength is independent of the (acute) angle between them, so that they prefer to be orthogonal to each other. However, unless $N \leq d+1$, it is impossible for all pairs of lines to be orthogonal. What then are their stable configurations? An unsolved conjecture of Fejes Toth (1959) asserts that the lines should be equidistributed as evenly as possible over an orthonormal basis in $\mathbf{R}^{d+1}$. By modifying the force to make it increase as a power of the distance, we show the analogous claim to be true for all positive powers if we are only interested in local stability, and for sufficiently large powers if we require global stability.

These results represent joint work with Tongseok Lim (of Purdue University's Krannert School of Management).