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*Monge solutions and uniqueness in multi-marginal optimal transport via graph theory*

In this talk, we will focus on the multi-marginal optimal transport problem with surplus  $b(x_1, \dots, x_m) = \sum_{\{i,j\} \in P} x_i \cdot x_j$ , where  $P \subseteq Q := \{\{i,j\} : i, j \in \{1, 2, \dots, m\}, i \neq j\}$ . We associate each surplus of this type with a graph with  $m$  vertices, whose set of edges is indexed by  $P$ . We then provide a natural reformulation of the problem in a graph theory approach, and establish uniqueness and Monge solution results for two general classes of surplus functions. In particular, these classes encapsulate the Gangbo and Świąch surplus and the surplus  $\sum_{i=1}^{m-1} x_i \cdot x_{i+1} + x_m \cdot x_1$ , whose origin lies in the time discretization of Arnold's variational interpretation of the incompressible Euler equation. This is joint work with Brendan Pass.