Nonlinear analysis on manifolds
Analyse non linéaire dans les variétés différentielles
(Org: **Siyuan Lu** (McMaster) and/et **Jérôme Vétois** (McGill))

SPYROS ALEXAKIS, University of Toronto
Singularity formation in Black hole interiors

Starting from classical examples of singularity formation inside black holes, I will recall the strong cosmic censorship conjecture of Penrose regarding question. I will also review some further predictions and known results on the generic behavior of the space-time metric as it terminates at a singularity; these results will be compared with the complementary picture on initial, big-bang type singularities. The main new result we will present is a recent proof of the perturbative stability of the Schwarzschild singularity in vacuum, under polarized perturbations of the initial data. The singularity that then forms is again of space-like character, and the solution displays asymptotically-velocity-term-dominated behavior upon approach to the singularity. Joint with G. Fournodavlos.

HUSSEIN CHEIKH-ALI, Université Libre de Bruxelles
The second best constant for the Hardy-Sobolev inequality on manifolds

We consider the second best constant in the Hardy-Sobolev inequality on a Riemannian manifold. More precisely, we are interested with the existence of extremal functions for this inequality. This problem was tackled by Djadli-Druet [1] for Sobolev inequalities. Here, we establish the corresponding result for the singular case. In addition, we perform a blow-up analysis of solutions to Hardy-Sobolev equations of minimizing type. This yields informations on the value of the second best constant in the related Riemannian functional inequality.

References

[1] Zindine Djadli and Olivier Druet, Extremal functions for optimal Sobolev inequalities on compact manifolds, Calc. Var. Partial Differential Equations **12** (2001), no. 1, 58–84.

EDWARD CHERNYSH, McGill University
A global compactness theorem for critical p-Laplace equations with weights

In this talk, we investigate the compactness of Palais-Smale sequences for a class of critical p-Laplace equations with weights. More precisely, we discuss a Struwe-type decomposition result for Palais-Smale sequences, thereby extending a recent result of Mercuri-Willem (2010) to weighted equations. In sharp contrast to the model case of the unweighted critical p-Laplace equation, all bubbling must occur at the origin. Furthermore, an adapted rescaling law is required to circumvent new difficulties introduced by the weights.

BEOMJUN CHOI, University of Toronto / KIAS
Liouville theorem for surfaces translating by sub-affine-critical powers of Gauss curvature

We construct and classify the translating solutions to the flows by sub-affine-critical powers of the Gauss curvature in $\mathbb{R}^3$. If α denotes the power, this corresponds to a Liouville theorem for degenerate Monge-Ampere equations $\det D^2u = (1+|Du|^2)^{2-\frac{1}{2\alpha}}$ on $\mathbb{R}^2$ for $0 < \alpha < 1/4$. For the affine-critical case $\det D^2u = 1$, a classical result by Jörgens, Calabi and Pogorelov shows the level curves of given solution are homothetic ellipses. In our case, the level curves converge asymptotically to a round circle or a curve with k-fold symmetry for some $3 \leq k \leq n_\alpha$. More precisely, these curves are closed shrinking curves to the $\frac{\alpha}{1-\alpha}$-curve shortening flow that were previously classified by Andrews in 2003. This is a joint work with K. Choi and S. Kim.

SHUBHAM DWIVEDI, Humboldt University, Berlin
Deformation theory of nearly G_2 *manifolds*

We will discuss the deformation theory of nearly G_2 manifolds. These are seven dimensional manifolds admitting real Killing spinors. After briefly discussing the preliminaries, we will show that the infinitesimal deformations of nearly G_2 structures are obstructed in general. Explicitly, we will show that the infinitesimal deformations of the homogeneous nearly G_2 structure on the Aloff–Wallach space are all obstructed to second order. We will also completely describe the cohomology of nearly G_2 manifolds. This talk is based on a joint work with Ragini Singhal (University of Waterloo) (https://arxiv.org/abs/2007.02497).

PENGFEI GUAN, McGill University
Locally constrained mean curvature type flows

The talk concerns a class of mean curvature type flows with constraints. The first of such flow involving mean curvature was considered in a previous joint work with Junfang Li to provide a flow approach to the classical isoperimetric inequality. Later, general fully nonlinear constrained flows were introduced for optimal geometric inequalities involving quermassintegrals. These flows are associated with variational properties of corresponding geometric quantities. We will discuss some recent results and open regularity problems.

ROBERT HASLHOFER, University of Toronto
Mean curvature flow through neck-singularities

In this talk, I will explain our recent work showing that mean curvature flow through neck-singularities is unique. The key is a classification result for ancient asymptotically cylindrical flows that describes all possible blowup limits near a neck-singularity. In particular, this confirms Ilmanen's mean-convex neighborhood conjecture, and more precisely gives a canonical neighborhood theorem for neck-singularities. Furthermore, assuming the multiplicity-one conjecture, we conclude that for embedded two-spheres mean curvature flow through singularities is well-posed. The two-dimensional case is joint work with Choi and Hershkovits, and the higher-dimensional case is joint with Choi, Hershkovits and White.

HAN HONG, University of British Columbia
Stability and index estiamtes of capillary surfaces

In this talk, we will discuss stability and index estimates for compact and noncompact capillary surfaces. A classical result in minimal surface theory says that a stable complete minimal surface in $\mathbb{R}^3$ must be a plane. We show that, under certain curvature assumptions, a strongly stable capillary surface in a 3-manifold with boundary has only three possible topological configurations. In particular, we prove that a strongly stable capillary surface in a half-space of $\mathbb{R}^3$ which is minimal or has the contact angle less than or equal to $\pi/2$ must be a half-plane. We also give index estimates for compact capillary surfaces in 3-manifolds by using harmonic one-forms. This is joint work with Aiex and Saturnino.

CHRISTOPHER KENNEDY, University of Toronto
A Bochner Formula on Path Space for the Ricci Flow

Aaron Naber (Northwestern) and Robert Haslhofer (Toronto) have characterized solutions of the Einstein equation $\mathrm{Rc}(g) = \lambda g$ in terms of both sharp gradient estimates for Brownian motion and a Bochner formula on elliptic path space PM. They also successfully characterized solutions of the Ricci flow $\partial_t g = -2\mathrm{Rc}(g)$ in terms of an infinite-dimensional gradient estimate on parabolic path space $P\mathcal{M}$ of space-time $\mathcal{M} = M \times [0, T]$.

In this talk, we shall generalize the classical Bochner formula for the heat flow on evolving manifolds $(M, g_t)_{t\in[0,T]}$ to an infinite-dimensional Bochner formula for martingales, thus proving the parabolic counterpart of recent results in the elliptic setting as well as characterizing solutions of the Ricci flow in terms of Bochner inequalities on parabolic path space. Time-permitting, we shall also discuss gradient and Hessian estimates for martingales on parabolic path space as well as a condensed proof of previous characterizations of the Ricci flow.

YANGYANG LI, Princeton University

Generic Regularity of Minimal Hypersurfaces in Dimension 8

The well-known Simons' cone suggests that minimal hypersurfaces could be possibly singular in a Riemannian manifold with dimension greater than 7, unlike the low dimensional case. Nevertheless, it was conjectured that one could perturb away these singularities generically. In this talk, I will discuss how to perturb them away to obtain a smooth minimal hypersurface in an 8-dimension closed manifold, by induction on the "capacity" of singular sets. This result generalizes the previous works by N. Smale and by Chodosh-Liokumovich-Spolaor to any 8-dimensional closed manifold. This talk is based on joint work with Zhihan Wang.

JIAWEI LIU, Otto-von-Guericke-University Magdeburg

Ricci flow starting from an embedded closed convex surface in $\mathbb{R}^3$

We talk about the existence and uniqueness of Ricci flow that admits an embedded closed convex surface in $\mathbb{R}^3$ as metric initial condition. The main point is a family of smooth Ricci flows starting from smooth convex surfaces whose metrics converge uniformly to the metric of the initial surface in intrinsic sense. This is joint work with Jiuzhou Huang.

HUSSEIN MESMAR, lorraine university IECL

Solution for Hardy-Sobolev equation in presence of isometrie

Let $(M;g)$ be a smooth compact Riemannian manifold of dimension $n \geq 4$, G a closed subgroup of the group of isometries $Isom_g(M)$ of (M,g) and $k = \min_{x\in M} dimGx$, where Gx denotes the orbit of a point $x \in M$ under G. We fixe a point $x_0 \in M$ that $dimGx_0 = k$ and $s \in (0;2)$. We say that a function $\phi : M \to \mathbb{R}$ is G-invariant if $\phi(gx) = \phi(x)$ for any $x \in M$ and $g \in G$. We investigate a suffcient condition for the existence of a distributional continuous positive G-invariant solution for the Hardy-Sobolev equation

$$\Delta_g u + au = \frac{u^{2^*(k,s)-1}}{d_g(x, Gx_0)^s} + hu^{q-1} \tag{E}$$

where $\Delta_g := -div_g(\nabla)$ is the Laplace-Beltrami operator, ,a, $h \in C^0(M)$, $h \geq 0$, d_g is the Riemannian distance on $(M;g)$, $2^*(k,s) = \frac{2(n-k-s)}{n-k-2}$ and $q \in (2, 2^*(k,s))$ with $2^* = 2^*(0,0)$. We prove that the existence of a Mountain Pass solution for the above perturbative equation depends only on the perturbation. For that we need to prove first that for any $\epsilon > 0$, exist $A > 0$ and $B_\epsilon = B(\epsilon) \geq 0$ so that for any $u \in L^{2^*(k,s)}(M, d_g(x, Gx_0)^{-s})$

$$||u||^2_{L^{2^*(k,s)}(M,d_g(x,Gx_0)^{-s})} \leq (A+\epsilon)||\nabla u||_2^2 + B_\epsilon ||u||_2^2$$

PENGZI MIAO, University of Miami

On interaction between scalar curvature and boundary mean curvature

Scalar curvature and mean curvature are some of the most basic curvature quantities associated to a Riemannian manifold and its hypersurfaces, respectively. In a relativistic context, scalar curvature relates to matter distribution in a spacetime and mean curvature is used to compute the quasi-local mass of a finite body. In Riemannian geometry, existence and non-existence of positive scalar curvature metrics is a fundamental question on closed manifolds. If the manifold is noncompact, important results on metrics with nonnegative scalar curvature include the Riemannian positive mass theorem and the Riemannian Penrose inequality. In this talk, we discuss how nonnegative scalar curvature in the interior of a compact manifold influences the mean curvature of its boundary hypersurface. Part of the talk is based on joint work with Siyuan Lu.

ALEX MRAMOR, Johns Hopkins University
On the unknottedness of self shrinkers

Self shrinkers are basic singularity models for the mean curvature flow. Much progress has been made in their study but outside some curvature convexity conditions and other special cases they are still not fully understood. In this talk I'll discuss some "unknottedness" results for self shrinkers in $\mathbb{R}^3$, which for instance imply that a self shrinking torus cannot be a tubular neighborhood of a nontrivial knot. The arguments discussed use the mean curvature flow and include some families of noncompact self shrinkers - closed self shrinkers were previously considered in a joint work with Shengwen Wang.

KEATON NAFF, Columbia University
A local noncollapsing estimate for mean curvature flow

We will discuss noncollapsing in mean curvature flow and prove a local version of the noncollapsing estimate. By combining our result with earlier work of X.-J. Wang, it follows that certain ancient convex solutions that sweep out the entire space are noncollapsed. This is joint work with S. Brendle.

JIEWON PARK, Caltech
The Laplace equation on noncompact Ricci-flat manifolds

We will discuss geometric applications of the Laplace equation on a complete Ricci-flat manifold with Euclidean volume growth. We will focus on how to identify two arbitrarily far apart scales in the manifold in a natural way, exploiting the Łojasiewicz inequality of Colding-Minicozzi, in the case when a tangent cone at infinity has smooth cross section. We also prove a matrix Harnack inequality for the Green function when there is an additional condition on sectional curvature, which is an analogue of various matrix Harnack inequalities obtained by Hamilton and Li-Cao in different time-dependent settings.

SÉBASTIEN PICARD, UBC
Topological Transitions of Calabi-Yau Threefolds

It was proposed in the works of Clemens, Reid and Friedman to connect Calabi-Yau threefolds with different topologies by a process known as a conifold transition. This operation may produce a non-Kahler complex manifold with trivial canonical bundle. In this talk, we will discuss the propagation of differential geometric structures such as metrics with special holonomy and Yang-Mills connections through conifold transitions. This is joint work with T. Collins and S.-T. Yau.

BRUNO PREMOSELLI, Université Libre de Bruxelles
Towers of bubbles for Yamabe-type equations in dimensions larger than 7

In this talk we consider perturbations of Yamabe-type equations on closed Riemannian manifolds. In dimensions larger than 7 and on locally conformally flat manifolds we construct blowing-up solutions that behave like towers of bubbles (or bubble-trees) concentrating at a critical point of the mass function. Our result does not assume any symmetry on the underlying manifold.

We perform our construction by combining finite-dimensional reduction methods with a linear blow-up analysis. Our approach works both in the positive and sign-changing case. As an application we prove the existence, on a generic bounded open set of $\mathbb{R}^n$, of blowing-up solutions of the Brézis-Nirenberg equation that behave like towers of bubbles with alternating signs.

FRÉDÉRIC ROBERT, Université de Lorraine
Blowing-up solutions for second-order critical elliptic equations: the impact of the scalar curvature

Given a closed manifold (M^n, g), $n \geq 3$, Olivier Druet proved that a necessary condition for the existence of energy-bounded blowing-up solutions to perturbations of the equation

$$\Delta_g u + h_0 u = u^{\frac{n+2}{n-2}},\ u > 0 \text{ in } M$$

is that $h_0 \in C^1(M)$ touches the Scalar curvature somewhere when $n \geq 4$ (the condition is different for $n = 6$). In this paper, we prove that Druet's condition is also sufficient provided we add its natural differentiable version. For $n \geq 6$, our arguments are local. For the low dimensions $n \in \{4, 5\}$, our proof requires the introduction of a suitable mass that is defined only where Druet's condition holds. This mass carries global information both on h_0 and (M, g).

XI SISI SHEN, Northwestern University
Estimates for metrics of constant Chern scalar curvature

We discuss the existence problem of constant Chern scalar curvature metrics on a compact complex manifold. We prove a priori estimates for these metrics conditional on an upper bound on the entropy, extending a recent result by Chen-Cheng in the Kähler setting.

VLADMIR SICCA, McGill University
A prescribed scalar and boundary mean curvature problem on compact manifolds with boundary

In this talk I will present our recent result in the problem of finding a metric in a given conformal class with prescribed nonpositive scalar curvature and nonpositive boundary mean curvature, on a compact manifold with boundary. We established a necessary and sufficient condition in terms of a conformal invariant that measures the zero set of the target curvatures, which we call the relative Yamabe invariant of the set. (This is a joint work with Gantumur Tsogtgerel).

FREID TONG, Columbia University
On the degenerations of asymptotically conical Calabi-Yau metrics

The analytic study of complete non-compact Ricci-flat Kahler metrics began with the work of Tian-Yau in the 90s, who used PDE methods to produce many interesting examples of such metrics. In this talk, we will discuss the degenerations of non-compact Ricci-flat Kahler metrics from an analytic point of view: by studying the limit of the corresponding complex Monge-Ampere equations. In certain cases, we will see that the degenerate metric limit induces a complete singular Ricci-flat Kahler metric on a quasi-projective variety and we will discuss the applications to constructions of complete Ricci-flat Kahler metrics with singularities. This is joint work with T. Collins and B. Guo.

JUNCHENG WEI, University of British Columbia
Sharp quantitative estimates for Struwe's decomposition

Suppose $u \in D^{1,2}(\mathbb{R}^n)$. In a fundamental paper in 1984, Struwe proved that if $||\Delta u + u^{\frac{2n}{n-2}}||_{H^{-1}} := \Gamma(u) \to 0$ then $\delta(u) \to 0$, where $\delta(u)$ denotes the $D^{1,2}(\mathbb{R}^n)$-distance of u from the manifold of sums of Talenti bubbles, i.e.

$$\delta(u) := \inf_{\substack{(z_1,\cdots,z_\nu)\in\mathbb{R}^n \\ \lambda_1,\cdots,\lambda_\nu>0}} \left\| \nabla u - \nabla \left(\sum_{i=1}^{\nu} U\left[z_i, \lambda_i\right] \right) \right\|_{L^2}.$$

In 2019, Figalli and Glaudo obtained the first quantitative version of Struwe's decomposition in lower dimensions, namely $\delta(u) \lesssim \Gamma(u)$ when $3 \leq n \leq 5$. In this talk, I will present the following quantitative estimates of Struwe's decomposition in higher dimensions:

$$\delta(u) \leq C \begin{cases} \Gamma(u)\,|\log \Gamma(u)|^{\frac{1}{2}} & \text{if } n = 6, \\ |\Gamma(u)|^{\frac{n+2}{2(n-2)}} & \text{if } n \geq 7. \end{cases}$$

Furthermore, we show that this inequality is sharp. (Joint work with B. Deng and L. Sun.)

FENGRUI YANG, McGill University

Prescribed curvature measure problem in hyperbolic space

The problem of the prescribed curvature measure is one of the important problems in differential geometry and nonlinear partial differential equations. In this talk, I will talk about prescribed curvature measure problem in hyperbolic space. We establish the existence and regularity of solutions to the problem. The key is the C^2 regularity estimates for solutions to the corresponding fully nonlinear PDE in the hyperbolic space.

SIYI ZHANG, University of Notre Dame

Conformally invariant rigidity theorems on four-manifolds with boundary

We introduce conformal and smooth invariants on oriented, compact four-manifolds with boundary and show that "positivity" conditions on these invariants will impose topological restrictions on underlying manifolds with boundary. We also establish conformally invariant rigidity theorems for Bach-flat four-manifolds with boundary under the assumptions on these invariants. It is noteworthy to point out that we rule out some examples arising from the study of closed manifolds in the setting of manifolds with umbilic boundary.

XIANGWEN ZHANG, UC Irvine

A geometric flow for Type IIA superstrings

The equations of flux compactifications of Type IIA superstrings were written down by Tomasiello and Tseng-Yau. To study these equations, we introduce a natural geometric flow on symplectic Calabi-Yau 6-manifolds. We prove the well-posedness of this flow and establish the Shi-type estimates which provides a criterion for the long time existence. As an application, we make use of our flow to find optimal almost complex structures on certain homogeneous symplectic half-flat manifolds. This is based on joint work with Fei, Phong and Picard.