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Sharp quantitative estimates for Struwe's decomposition

Suppose $u \in D^{1,2}(\mathbb{R}^n)$. In a fundamental paper in 1984, Struwe proved that if $||\Delta u + u^{\frac{2n}{n-2}}||_{H^{-1}} := \Gamma(u) \to 0$ then $\delta(u) \to 0$, where $\delta(u)$ denotes the $D^{1,2}(\mathbb{R}^n)$-distance of u from the manifold of sums of Talenti bubbles, i.e.

$$\delta(u) := \inf_{\substack{(z_1,\cdots,z_\nu)\in\mathbb{R}^n \\ \lambda_1,\cdots,\lambda_\nu>0}} \left\| \nabla u - \nabla\left(\sum_{i=1}^{\nu} U\left[z_i,\lambda_i\right]\right)\right\|_{L^2}.$$

In 2019, Figalli and Glaudo obtained the first quantitative version of Struwe's decomposition in lower dimensions, namely $\delta(u) \lesssim \Gamma(u)$ when $3 \le n \le 5$. In this talk, I will present the following quantitative estimates of Struwe's decomposition in higher dimensions:

$$\delta(u) \le C \begin{cases} \Gamma(u)\left|\log\Gamma(u)\right|^{\frac{1}{2}} & \text{if } n=6, \\ |\Gamma(u)|^{\frac{n+2}{2(n-2)}} & \text{if } n\ge 7. \end{cases}$$

Furthermore, we show that this inequality is sharp. (Joint work with B. Deng and L. Sun.)