Mathematical models for ecological dynamics
Modèles mathématiques en dynamiques écologiques
(Org: **Frithjof Lutscher** (Ottawa) and/et **Olga Vasilyeva** (Memorial))

CHRISTINA COBBOLD, University of Glasgow
Impacts of range shifts for partially sedentary populations

Climate change is inducing range shifts for many species. In order to survive species must adapt or move to keep pace with their shifting range, but what does this mean for populations that are partially sedentary, with only a fraction of the population able to disperse? In this talk we address this question using integrodifference equations. Using a combination of stability analysis and numerical simulation we show that, provided climate velocity is not too large, partially sedentary populations can outperform fully dispersing populations in one of two ways: (i) by persisting at climate speeds where a fully dispersing population cannot, and (ii) exhibiting higher population densities.

MOHAMMAD EL SMAILY, University of Northern British Columbia
Asymptotics and spectral properties of an integrodifference model with a discontinuous kernel

In this talk, we will show some analytic results we obtained for an integral equation modelling the discrete time dynamics of a population in a patchy landscape. Mathematically, the patchiness in the habitat is reflected in the discontinuity of the kernel of the integral operator, at a finite number of points in the whole domain. We prove that existence and uniqueness of a stationary state under certain assumptions on the principal eigenvalue of the linearized integral operator and the growth term as well. Under certain conditions the population undergoes extinction (in which case the stationary solution is 0 everywhere). This talk is from a joint work with Omar Abdul Halim.

SAMUEL FISCHER, Helmholtz Centre for Environmental Research – UFZ, Department of Ecological Modelling
Boosting propagule transport models with individual-specific data from mobile apps

Human traffic is an important vector for various invasive species and infectious diseases. Hence, modelling the dynamics of these species and diseases requires accurate estimates of human traffic flows. Such estimates are often computed using traffic models fitted to on-site or mail-out survey data. Recently, data collected via mobile apps have become a promising alternative, potentially allowing for more intricate traffic models incorporating numerous covariates and accounting for vectors' individual preferences. However, as potential vectors may not record all their trips, data voluntarily recorded via apps come with an additional level of uncertainty. We show how the benefits of app-based data can be exploited despite this drawback by accounting for repeating behaviour of vectors. We demonstrate our approach by considering a case study estimating angler traffic in Alberta, where anglers facilitate the spread of a parasite-induced fish disease. Our results do not only provide valuable insights into the traffic patterns of anglers in Alberta but also indicate that anglers' local preferences and their tendency to revisit previous destinations significantly affect traffic volumes between waterbodies. Ignoring these individual characteristics could lead to significant overestimates of vector traffic and propagule dispersal.

This work is joint with Pouria Ramazi, Sean Simmons, Mark Poesch, and Mark Lewis.

SANA JAHEDI, University of New Brunswick
The equations of nature and the nature of equations

Systems of N equations in N unknowns are ubiquitous in mathematical modelling. These systems, often nonlinear, are used to identify equilibria of dynamical systems in ecology, genomics, control, and many other areas. Structured systems, where the variables that are allowed to appear in each equation are pre-specified, are especially common. For modeling purposes, there is a great interest in determining circumstances under which physical solutions exist, even if the coefficients in the model equations are only approximately known.

The structure of a system of equations can be described by a directed graph G that reflects the dependence of one variable on another, and we can consider the family $\mathcal{F}(G)$ of systems that respect G. We define a solution X of $F(X) = 0$ to be robust if for each continuous F^* sufficiently close to F, a solution X^* exists. Robust solutions are those that are expected to be found in real systems. There is a useful concept in graph theory called "cycle-coverable". We show that if G is cycle-coverable, then for "almost every" $F \in \mathcal{F}(G)$ in the sense of prevalence, every solution is robust. Conversely, when G fails to be cycle-coverable, each system $F \in \mathcal{F}(G)$ has no robust solutions.

Failure to be cycle-coverable happens precisely when there is a configuration of nodes that we call a "bottleneck," a criterion that can be verified from the graph. A "bottleneck" is a direct extension of what ecologists call the Competitive Exclusion Principle, but we apply it to all structured systems.

YU JIN, University of Nebraska-Lincoln
Population dynamics in river networks

Natural rivers connect to each other to form networks. The geometric structure of a river network can significantly influence spatial dynamics of populations in the system. We consider a process-oriented model to describe population dynamics in river networks of trees, establish the fundamental theories of the corresponding parabolic problems and elliptic problems, derive the persistence threshold by using the principal eigenvalue of the eigenvalue problem, and define the net reproductive rate to describe population persistence or extinction. By virtue of theoretical and numerical analyses, we investigate the effects of biotic and abiotic factors, especially the structure of the river network, the diffusion rate, and the flow velocity on population persistence in temporally constant or fluctuating environments.

MARK LEWIS, University of Alberta
Inside Dynamics for Integrodifference Equations

In this talk I will discuss recent modelling and analysis of integrodifference equation models for the asymptotic genetic structure of populations undergoing range expansion. To analyze the genetic consequences for long term population spread, we decompose the solution into neutral genetic components called neutral fractions. The "inside dynamics" then describe the spatiotemporal evolution of these neutral fractions. Extensions are made to include stage-structure in the population dynamics and mutations in the genetic fractions. This work is joint with Nathan Marculis and Roger Lui.

FRITHJOF LUTSCHER, University of Ottawa
Transient dynamics for equilibrium and non-equilibrium communities

Asymptotically stable states continue to be the subject of study in most dynamical systems models in biology. However, true convergence to such states is rare in real systems. For example, human activities or natural events may perturb locally stable equilibrium communities. The study of transient dynamics attempts to gain information about the qualitative behaviour of dynamical systems before an asymptotically stable state is reached.

One particular question of transient dynamics asks how long a biological community will take to return to a stable steady state after a perturbation and how "far" from that state it may get in the process. To answer those questions, researchers have defined the "resilience" and "reactivity" of a system. For an appropriate choice of norms, these quantities can be measured in terms of eigenvalues of certain matrices. I will first review these measures and discuss some of the links to matrix analysis. Then I will suggest possible extensions of the theory to periodically forced systems and periodic orbits in autonomous systems and examine some of their properties.

MARIA MARTIGNONI, Memorial University
Mathematical insights into mechanisms leading to coexistence and competitive exclusion among mutualist guilds

Mutualistic interactions are gaining increasing attention in the scientific literature, especially as pollination and plant-microbe symbioses play a key role in agricultural productivity. In particular, the widespread symbiosis between plants and arbuscular mycorrhizal (AM) fungi, offers a promising sustainable alternative for maintaining productivity in farmland. Despite the potential benefits for soil quality and crop yield associated with the use of AM fungi, experiments assessing the effective establishment of the fungi in the field have given inconsistent results. Additionally, it is not clear whether the introduction of commercial AM fungi could lead to a biodiversity loss in the native fungal community, and ultimately have a negative impact on plant growth. We developed a series of mathematical models for plant and AM fungal growth to assess the establishment, spread and impact of an introduced species of AM fungi on the native fungal community and on plant productivity. Our models provide a theoretical framework to determine the circumstances under which the inoculated fungal species can coexist with the native fungal community and effectively boost productivity, versus when inoculation constitutes a biodiversity risk and, ultimately, a detriment to crop yield. Overall, our results show that diversity within mutualistic communities promotes productivity and reduces the risk of invasion and biodiversity loss posed by the introduction of a less mutualistic, or even parasitic, species. Although my analysis focuses on plant-fungal interactions, my findings provide valuable criteria to assess the impact of species introduction in mutualistic communities in general, such as other beneficial microbes or pollinator communities.

REBECCA TYSON, University of British Columbia Okanagan
Phase-sensitive tipping: New mechanism for extinction

Global change is expected to lead to climate changes that include greater amplitudes and longer "periods" in climate variability. Many recent studies have noted that the greater variability associated with global change often has more impact than the change in average behaviour (temperature, precipitation, etc). In this paper we explore how changes in climate variability could interact with a system that is already oscillating, namely, predator-prey systems. We include an Allee effect in the prey equation so that we can determine whether or not extinction is deterministically possible, simply as a result of climatic variability. We find that variability-induced extinction is possible for both the Rosenzweig-MacArthur (RM) and Leslie-Gower-May (LGM) model systems and for realistic parameter values for the canada lynx and snowshoe hare.

Joint work with Hassan Alkhayuon, and Sebastian Wieczorek

OLGA VASILYEVA, Grenfell Campus, Memorial University of Newfoundland
Steady states of nonlinear reaction-diffusion-advection models: phase plane approach

Steady states of nonlinear reaction-diffusion-advection (RDA) models can be viewed as solutions of a system of two first order ODEs (subject to appropriate boundary conditions). Geometrically, they are represented by orbits in the phase plane, generated by the corresponding flow operator. In the basic case of a logistic RDA model describing population dynamics in a finite river segment, the phase plane approach helped to establish the existence and uniqueness of a positive steady state solution for sufficiently low advection speeds and sufficiently large domains. In this talk, I will discuss applications of the phase plane technique in two extensions of this basic model. In one setting, we increase the complexity of the habitat by considering a binary river network. In the second setting, we increase the complexity of the reaction term. Namely, we study an extension of the classical spatial spruce budworm (SBW) model (where reaction term accounts for predation), with advection term describing biased movement of larvae due to prevailing winds. In the river network case, the phase plane approach helps us to find conditions for existence and uniqueness of positive steady state. In the SBW model, we use phase plane analysis to determine the conditions for existence of the outbreak solutions. In particular, we observe that increasing advection can prevent outbreaks while allowing persistence in form of an endemic state. We obtain upper and lower bounds for the critical advection for outbreaks.

HAO WANG, University of Alberta
Optimal foraging strategies

Nutritional constraints are common as food resources are rarely optimally suited for grazing species. Elemental mismatches between trophic levels can influence population growth and foraging behaviors. Grazing species, such as Daphnia, utilize optimal foraging techniques, such as compensatory feeding. Here, we develop two stoichiometric producer–grazer models, a base model that incorporates a fixed energetic foraging cost and an optimal foraging model where energetic foraging costs depend on food nutritional content. A variable energetic foraging cost results in cell quota-dependent predation behaviors.

Analyzing and comparing these two models allows us to investigate the potential benefits of stoichiometric compensatory foraging behaviors on grazer populations. Optimal foraging strategies depend on environmental conditions, such as light and nutrient availability. In low-light conditions, fixed energetic foraging appears optimal regardless of the nutrient loads. However, in higher light conditions and intermediate nutrient loads, grazers utilizing compensatory foraging strategies gain an advantage. Overall, grazers can benefit from compensatory feeding behaviors when the food nutrient content of their prey becomes low or high. At the end of the talk, I will briefly mention a discrete-time version in comparison with the continuous-time version.

XIAOYING WANG, Trent University

How spatial heterogeneity affects transient behavior in reaction-diffusion systems for ecological interactions

Most studies of ecological interactions study asymptotic behavior, such as steady states and limit cycles. The transient behavior, i.e., qualitative aspects of solutions as and before they approach their asymptotic state, may differ significantly from asymptotic behavior. Understanding transient dynamics is crucial to predicting ecosystem responses to perturbations on short time scales. Several quantities have been proposed to measure transient dynamics in systems of ordinary differential equations. Here, we generalize these measures to reaction-diffusion systems in a rigorous way and prove various relations between the non-spatial and spatial effects, as well as an upper bound for transients. This extension of existing theory is crucial for studying how spatially heterogeneous perturbations and the movement of biological species involved affect transient behaviors. We illustrate several such effects with numerical simulations.

XINGFU ZOU, University of Western Ontario

Spatial-Temporal dynamics of diffusive Lotka-Volterra competition model with a shifting habitat

In this talk, I will report some recent results on the Spatial-temporal dynamics of some diffusive Lotka-Volterra competition models in an environment that worsens at a constant speed. Both random diffusion and nonlocal diffusion will be considered. Conditions for a species to be persistent or extinct, as well as the patterns of persistence and extinction will be discussed.