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On the finiteness of strong maximal functions associated to functions whose integrals are strongly differentiable

Besicovitch proved that if f is an integrable function on $\mathbb{R}^2$ whose associated strong maximal function $M_S f$ is finite a.e., then the integral of f is strongly differentiable. On the other hand, Papoulis proved the existence of a function in $L^1(\mathbb{R}^2)$ (taking on both positive and negative values) whose integral is strongly differentiable but whose associated strong maximal function is infinite on a set of positive measure. In this talk, we discuss a recent result of Hagelstein and Oniani that if $f \in L^1(\mathbb{R}^n)$ is a *nonnegative* function whose integral is strongly differentiable and moreover such that $f(1+\log^+ f)^{n-2}$ is integrable, then $M_S f$ is finite a.e. This result is sharp in that, if ϕ is a convex increasing function on $[0,\infty)$ such that $\phi(0)=0$ and with $\phi(u) = o(u(1+\log^+ u)^{n-2})$ $(u \to \infty)$, then there exists a nonnegative function f on $\mathbb{R}^n$ such that $\phi(f)$ is integrable on $\mathbb{R}^n$ and the integral of f is strongly differentiable, although $M_S f$ is infinite on a set of positive measure.