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Robust solutions in systems of equations

Joint work with Sana Jahedi and Tim Sauer.

We begin by asking what would we like (really smart) high-school students to know about systems of equations. A structured system of equations $F(x) = c$ where $F : \mathbb{R}^N \to \mathbb{R}^M$ is a system of M equations in which it is specified which of the N variables are allowed to appear in each equation. They are ubiquitous in mathematical modeling. Our goal in this article is to describe the global properties of solutions for structured systems of C^∞ functions. We describe general properties of solutions that follow from the system structure rather than from the particular details of the system. An important question when modeling natural systems is whether solutions are robust to small changes in the model. To address this question, we say F is "flat" if there exists an integer k such that for almost every x_0, the set $F^{-1}F(x_0)$ of solutions x of $F(x) = F(x_0)$ is a k-dimensional manifold. When $k = 0$, the solutions are isolated points. Systems of polynomials are examples of flat functions. But there are C^∞ functions F that are not flat, even when $N = M = 1$. We state conditions on vector spaces of C^∞ functions that imply that "almost every" (in the sense of prevalence) F in the vector space is k-flat for some k, and we show that if $k = M - N$ almost every F has the property that almost every x is a robust solution of $F(x) = c$ for some c. Then x is robust-to-small-changes in c and F.

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