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Steiner's problem ... Bussey's solution

A set-system of order N is a pair $(X, \mathcal{B})$, where X is N-element set of $points$ and $\mathcal{B}$ is a collection of subsets of X called $blocks$.

In 1852, Professor Dr. J. Steiner of Berlin, asked for which number N does there exist a set system containing no pairs that has order N and maximum block size k satisfying

1. no block properly contains another block, and
2. for all $t = 2, 3, ..., k-1$ every t-set that does not contain a block is contained in exactly one block of size $(t+1)$.

The only known solution with maximum block size at least 5 was an infinite family exhibited by W.H. Bussey from the University of Minnesota in 1914. He provides a construction for each $k \geq 5$ a set-system of order $N = 2^{k-1} - 1$ and maximum block size k satisfying Steiner's conditions. In 1984, H. Hanani, apparently unaware of Bussey's solution, gives exactly the same solution.

In this talk I will discuss Bussey's solution and report on the progress that Charlie Colbourn, Patric Östegård and I have made in constructing another solution.