GAVIN OROK, University of Waterloo

Local Dimensions of Self-Similar Measures in the Sierpinski Gasket

This project attempted to determine the set of all possible local dimensions of points in the support of probability measures in the iterated function system (IFS) that generates the Sierpinski Gasket fractal. The Sierpinski Gasket is the generalization of the Sierpsinski's Triangle fractal to the case where the contraction factor of the IFS is in $(\frac{1}{2}, \frac{2}{3})$. Calculations of the fractal's Hausdorff dimension and the local dimensions of measures are challenging in this case because there are overlaps between the triangular images from the IFS. The method used in this project to overcome difficulties caused by these overlaps was based on work by Kathryn Hare, Kevin Hare and Kevin Matthews that studied the one-dimensional version of this problem, the Bernoulli convolutions. Like in this work, the analysis focused on multinacci number contraction factors that resulted in the measures used being of finite type. First, a system of finitely many characteristic vectors was constructed that partition the images from the IFS into sets with disjoint interior. Then, a system of transition matrices was derived that relate the measures of consecutive parents and children in admissible paths through the characteristic vectors. Products of these matrices were used in place of closed balls to simplify calculations of local dimensions. Finally, these simplified calculations were analyzed over possible paths of characteristic vectors and points in the support to determine bounds on the local dimensions. Overall, the research managed to find trends in heuristic upper and lower bounds on the set of local dimensions.