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On the renewal theorem for maxima on trees

We consider the distributional fixed-point equation:

$$R \stackrel{\mathcal{D}}{=} Q \vee \left(\bigvee_{i=1}^{N} C_i R_i \right),$$

where the $\{R_i\}$ are i.i.d. copies of R, independent of the vector $(Q, N, \{C_i\})$, where $N \in \mathbb{N}$, $Q, \{C_i\} \geq 0$ and $P(Q > 0) > 0$. By setting $W = \log R$, $X_i = \log C_i$, $Y = \log Q$ it is equivalent to the high-order Lindley equation

$$W \stackrel{\mathcal{D}}{=} \max \left\{ Y, \max_{1 \leq i \leq N} (X_i + W_i) \right\}.$$

It is known that under Kesten assumptions,

$$P(W > t) \sim H e^{-\alpha t}, \qquad t \to \infty,$$

where $\alpha > 0$ solves the Cramér-Lundberg equation $E\left[\sum_{j=1}^{N} C_i^{\alpha}\right] = E\left[\sum_{i=1}^{N} e^{\alpha X_i}\right] = 1$. The main goal of this paper is to provide an explicit representation for $P(W > t)$, which can be directly connected to the underlying weighted branching process where W is constructed and that can be used to construct unbiased and strongly efficient estimators for all t. Furthermore, we show how this new representation can be directly analyzed using Alsmeyer's Markov renewal theorem, yielding an alternative representation for the constant H. We provide numerical examples illustrating the use of this new algorithm. This is a joint work with Bojan Basrak, Michael Conroy and Mariana Olvera-Cravioto.