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On a conjecture of Darmon–Rotger in the adjoint CM case

Let E be an elliptic curve over $\mathbf{Q}$ such that $L(E,s)$ has sign $+1$ and vanishes at $s=1$, and let $p>3$ be a prime of good ordinary reduction for E. A construction of Darmon–Rotger attaches to E, and an auxiliary weight one cuspidal eigenform g such that $L(E,\mathrm{ad}^0(g),1)\neq 0$, a Selmer class $\kappa_p(E,g,g^*)\in \mathrm{Sel}(\mathbf{Q},V_pE)$. They conjectured that the following are equivalent: (1) $\kappa_p(E,g,g^*)\neq 0$, (2) $\dim_{\mathbf{Q}_p}\mathrm{Sel}(\mathbf{Q},V_pE)=2$.

In this talk I will outline a proof of Darmon–Rotger's conjecture when g has CM and $\mathrm{Sha}(E/\mathbf{Q})[p^\infty]<\infty$ (and some mild additional hypotheses). If time permits, I'll also say a few words about the ongoing extension of these results to the case of supersingular primes p. Based on joint work with Ming-Lun Hsieh.