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Fields generated by points on superelliptic curves

We give an asymptotic lower bound on the number of field extensions generated by algebraic points on superelliptic curves over $\mathbb{Q}$ with fixed degree n, discriminant bounded by X, and Galois closure S_n. For C a fixed curve given by an affine equation $y^m = f(x)$ where $m \geq 2$ and deg $f(x) = d \geq m$, we find that for all degrees n divisible by gcd(m, d) and sufficiently large, the number of such fields is asymptotically bounded below by X^{c_n}, where $c_n \to 1/m^2$ as $n \to \infty$. This bound is determined explicitly by parameterizing x and y by rational functions, counting specializations, and accounting for multiplicity. We then give geometric heuristics suggesting that for n not divisible by gcd(m, d), degree n points may be less abundant than those for which n is divisible by gcd(m, d). Namely, we discuss the obvious geometric sources from which we expect to find points on C and discuss the relationship between these sources and our parametrization. When one a priori has a point on C of degree not divisible by gcd(m, d), we argue that a similar counting argument applies. This talk is based on joint work with Christopher Keyes.