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On a problem of Graham, Erdos, and Pomerance on the p-divisibility of central binomial coefficients

This is joint work with Hamed Mousavi and Maxie Schmidt. We show that for any set of $r \geq 1$ sufficiently large primes $p_1, ..., p_r$, there are infinitely many integers n, such that $\binom{2n}{n}$ is divisible by these primes with multiplicity of size at most $o(\log n)$. This is equivalent to saying we can find integers n whose base p_1, base p_2, ..., and base p_r expansions all simultaneously have almost all their digits "small". Doing this for 2 primes at once (the case r=2) is not difficult (Erdos proved this version); but it is significantly more challenging to prove it for $r \geq 3$; in fact, Graham offered a large sum of money – and considered it to be one of his favorite problems – to solve the case r=3 for the primes $3, 5,$ and 7. Our proof involves bypassing a deep, unsolved problem in diophanine approximation and algebraic number theory, called Schanuel's Conjecture, through the use of a number of methods from analytic number theory and additive combinatorics (and properties of generalized Vandermonde and totally positive matrices).