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A unified approach to operator monotone functions

The notion of operator monotonicity dates back to a work by Löwner in 1934. A map $F : S^n \to S^m$ is called *operator monotone*, if $A \succeq B$ implies $F(A) \succeq F(B)$. (Here, S^n is the space of symmetric matrices with the semidefinite partial order $\succeq$.) Often, the function F is defined in terms of an underlying simpler function f. Of main interest is to find the properties of f that characterize operator monotonicity of F. In that case, it is said that f is also operator monotone. Classical examples are the Löwner operators and the spectral (scalar-valued isotropic) functions. Operator monotonicity for these two classes of functions is characterized in seemingly very different ways.

This talk extends the notion of operator monotonicity to symmetric functions f on k arguments. The latter is used to define *(generated) k-isotropic maps* $F : S^n \to S^{\binom{n}{k}}$ for any $n \geq k$. Necessary and sufficient conditions are given for f to generate an operator monotone k-isotropic map F. When $k = 1$, the k-isotropic map becomes a Löwner operator and when $k = n$ it becomes a spectral function. This allows us to reconcile and explain the differences between the conditions for monotonicity for the Löwner operators and the spectral functions.