Recent advances in theory and applications of functional differential equations
Progrès récents dans la théorie et les applications des équations différentielles fonctionnelles
(Org: **Victor Leblanc** (Ottawa))

KEVIN CHURCH, McGill University

Computer-assisted proof of Hopf bifurcation in functional differential equations of mixed type

I will present a computational approach to Hopf bifurcation verification for functional differential equations of mixed type. The verification of a steady state, imaginary eigenvalues and their transversality amounts to a finite-dimensional problem which we rigorously solve using a Newton-Kantorovich-type theorem. To prove the imaginary eigenvalues are simple and that there is no resonance, we use some a priori estimates and rigorous contour integration of the characteristic equation to count all eigenvalues in a neighbourhood of the imaginary axis. As an application, we prove some results on periodic traveling waves in the Fisher equation with a nonlocal reaction term. This is joint with with Jean-Philippe Lessard.

TERESA FARIA, University of Lisbon

Stability for nonautonomous linear delayed differential systems

We study the stability of general nonautonomous linear differential equations with infinite delays. Delay independent criteria, as well as criteria depending on the size of bounded diagonal delays are established. Our results encompass DDEs with discrete and distributed delays, and enhance some recent achievements in the literature.

VICTOR LEBLANC, University of Ottawa

Degenerate Hopf Bifurcation in DDEs and Endemic Bubbles

We consider 2-parameter families of retarded functional differential equations (RFDE) which undergo Hopf bifurcation from an equilibrium, but for which the crossing condition of the Hopf theorem is violated (such a degeneracy is codimension 2). We classify the possible bifurcation diagrams in terms of the nonlinearities of the RFDE, and we apply the results to an SIS disease model incorporating delayed behavioral response.

ANDRÉ LONGTIN, University of Ottawa

Non-monotonic complexity with increasing numbers of delays

We investigate transitions to simple dynamics in first-order nonlinear differential equations with multiple delays. Multiple delays can destabilize fixed points and promote high-dimensional chaos, but many delays can also induce stabilization onto simpler dynamics. We focus on this behaviour as a function of the number of delays. Dynamical complexity is shown to depend on the precise distribution of delays. A narrow spacing between individual delays favours chaotic behaviour, while a lower density of delays enables stable periodic or fixed point behaviour. During complexity decrease, the number of roots of the characteristic equation around the fixed point that have a positive real part decreases. These roots behave in fact in a similar manner to the Lyapunov exponents and the Kolmogorov-Sinai entropy for these multi-delay systems, and can thus serve as a proxy for those dynamical invariants. Our results rely on a novel method to estimate the Lyapunov spectrum of multi-delay nonlinear systems, as well as on permutation entropy computations. Surprisingly, complexity collapse upon adding more delays can occur abruptly through an inverse period-doubling sequence. Our results shed light on the dynamical effects of the transition from discrete to continuous delay distributions. We also discuss the implications of our results for reservoir computing.

MICHAEL MACKEY, McGill University

State dependent delays induce novel dynamics in gene regulatory systems

This talk will review models for the bacterial regulation of gene expression and function for both repressible (negative feedback) and inducible (positive feedback) genes, and the nature of the nonlinearities involved. I argue that both the delays due to transcription of DNA to mRNA and translation of mRNA to produce protein are likely state dependent. The consequences of this turn out to be relatively astonishing in the sense that the state dependence of these delays can lead to completely new dynamical behaviors that are not present when the delays are constant. In both inducible and repressible systems the state dependence in the delays may lead to the appearance of more steady states as well as unexpected bifurcations not present when the delays are constant.

This is joint work with T. Gedon, A. Humphries, H.-O Walther, and Z. Wang.

FELICIA MAGPANTAY, Queen's University
Lyapunov-Razumikhin techniques for state-dependent delay differential equations

We present theorems for the Lyapunov and asymptotic stability of the steady state solutions to general state-dependent delay differential equations (DDEs) using Lyapunov-Razumikhin methods. These theorems build upon the previous work of Hale and Verduyn Lunel (1993), and Barnea (1969) which were mainly aimed at equations with simpler delay terms (e.g. constant and time-dependent delays), and are less applicable to state-dependent DDEs such as the following model equation,

$$\dot{u}(t) = \mu u(t) + \sigma u(t - a - cu(t)).$$

The stability region $\Sigma_\star$ of the zero solution to this model problem is known, and it is the same for both the constant delay ($c = 0$) and state-dependent delay ($c \neq 0$) cases. Using our results we can prove the asymptotic stability of the zero solution to this model problem in parts of $\Sigma_\star$, considerably expanding upon the work of Barnea who proved Lyapunov stability for the simpler $\mu = c = 0$ constant delay case. Similar techniques are used to derive a condition for global asymptotic stability of the zero solution to the model problem, and bounds on periodic solutions when the zero solution is unstable. This is joint work with A.R. Humphries

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CONNELL MCCLUSKEY, Wilfrid Laurier University
Modelling the growth of variants

There is a slow growth in the number of variants of concern for COVID-19. We model this growth as proportional to the number of infected individuals worldwide. Once new variants appear, they contribute to the spread.

Let $M(t)$ be the number of variants, and let $i(t.m)$ be the number of individuals infected with variant m at time t. Then

$$\frac{dM}{dt}(t) = \int_{m=0}^{M(t)} p(m)\, i(t,m)\, dm,$$

where $p(m)$ is the rate at which variant m slowly produces new variants.

What can we do with it? What impact do vaccines have on $M(t)$?

GAIL WOLKOWICZ, McMaster University
A Decay-Consistent Model of Population Growth and Competition with Delay

We derive an alternative expression for a delayed logistic equation in which the rate of change in the population involves a growth rate that depends on the population density during an earlier time period. In our formulation, the delay in the growth term is consistent with the rate of instantaneous decline in the population given by the model. Our formulation is a modification of [Arino et al., J. Theoret. Biol. 241(1):109–119, 2006] by taking the intraspecific competition between the adults and juveniles into account. We provide a complete global analysis showing that no sustained oscillations are possible. A threshold giving the interface between extinction and survival is determined in terms of parameters in the model. Our approach for analyzing the global dynamics incorporates the theory of chain transitive sets and the comparison theorem for cooperative delay differential equations. We extend our delayed logistic equation to a system modeling the competition of two species. For the competition model we provide results on local stability, bifurcation diagrams, and adaptive dynamics. Assuming that the species with shorter delay produces fewer newborns than the species with longer delay, we show that there is a critical value τ^* such that the evolutionary trend is to take the delay as close to τ^* as possible.

This is joint work with Chiu-Ju Lin and Ting-Hao Hsu

HUAIPING ZHU, York University

Models with delays for the transmission and control of COVID-19

The ongoing global COVID-19 epidemic poses a huge threat to human wellbeing and public health, waves of outbreaks continue to surge one year after it was declared as a global pandemic. There have been extensive modeling studies for the transmission which have been contributing greatly to inform decision-making. In this talk, I will discuss the role of delays in the modeling and dynamics for COVID-19 and their application in informing decision making. It is well-known that dynamical models with or without delays generate complex asymptotic dynamics through bifurcations. Usually there are two types of delays: one is due to the development or evolution of the virus, like the latency and incubation delays; the other is human but non-viral related, like delays in tracing and testing, quarantine and isolation, treatment due to lack of medical and health resources, or delays due to other social and behaviors in response to the control strategies. I will present examples of modeling with the two types of delays in control and mitigation of the epidemics of COVID-19 with application to inform rapid decision making. Examples of Wuhan and other selected cities will be presented.

XINGFU ZOU, University of Western Ontario

On a predator-prey system with digestion delay and anti-predation strategy

In this talk, I will present a predator-prey model incorporated with both cost and benefit from the prey's anti-predation response, together with a time delay in the transfer of biomass from the prey to the predator after predation. The model is a system of delay differential equations (DDEs). By analyzing this nonlinear DDE system, we obtain some insights on how the anti-predation response level (indirect effect) and the biomass transfer delay jointly affect the population dynamics; particularly we show how the nonlinearity in the predation term mediated by the fear effect affects the long term dynamics of the model system. These results seem to suggest a need to revisit some existing predator-prey models in the literature by incorporating the indirect effect and biomass transfer delay.