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The unreasonable effectiveness of p-adic methods in number theory

In a non-archimedean world, two subjects who walk away from each other, eager perhaps to observe current social distancing guidelines, might never succeed in coming more than one meter apart. Even worse, each successive step risks bringing them closer to each other than they ever were before, in a sequence of regularly spaced but increasingly egregious violations of sanitary rules. In spite of its paradoxical properties, the p-adic distance in such a world is well suited to number theory questions based on studying congruences, and forms the basis for an analysis that is just as rich and subtle as its real and complex counterparts. Some of the most central open problems in number theory, such as the Riemann hypothesis, the Birch and Swinnerton-Dyer conjecture, and the Stark conjecture admit p-adic variants which, far from being variants of concern, are significantly more tractable than their archimedean precursors. This lecture will describe the notable progress that has been achieved on these variants in the last decades, and attempt to explain what makes the progress possible.