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Solution for Hardy-Sobolev equation in presence of isometrie

Let $(M;g)$ be a smooth compact Riemannian manifold of dimension $n \geq 4$, G a closed subgroup of the group of isometries $Isom_g(M)$ of (M,g) and $k = \min_{x\in M} dimGx$, where Gx denotes the orbit of a point $x \in M$ under G. We fixe a point $x_0 \in M$ that $dimGx_0 = k$ and $s \in (0;2)$. We say that a function $\phi : M \to \mathbb{R}$ is G-invariant if $\phi(gx) = \phi(x)$ for any $x \in M$ and $g \in G$. We investigate a suffcient condition for the existence of a distributional continuous positive G-invariant solution for the Hardy-Sobolev equation

$$\Delta_g u + au = \frac{u^{2^*(k,s)-1}}{d_g(x, Gx_0)^s} + hu^{q-1} \tag{E}$$

where $\Delta_g := -div_g(\nabla)$ is the Laplace-Beltrami operator, ,a, $h \in C^0(M)$, $h \geq 0$, d_g is the Riemannian distance on $(M;g)$, $2^*(k,s) = \frac{2(n-k-s)}{n-k-2}$ and $q \in (2, 2^*(k,s))$ with $2^* = 2^*(0,0)$. We prove that the existence of a Mountain Pass solution for the above perturbative equation depends only on the perturbation. For that we need to prove first that for any $\epsilon > 0$, exist $A > 0$ and $B_\epsilon = B(\epsilon) \geq 0$ so that for any $u \in L^{2^*(k,s)}(M, d_g(x, Gx_0)^{-s})$

$$||u||^2_{L^{2^*(k,s)}(M,d_g(x,Gx_0)^{-s})} \leq (A+\epsilon)||\nabla u||_2^2 + B_\epsilon ||u||_2^2$$