General Relativity
Relativité générale
(Org: **Spyros Alexakis** and/et **Stefanos Aretakis** (Toronto))

NATHAN CARRUTH, University of Toronto/BIMSA
Highly localised gravitational waves in polarised translational symmetry

We discuss results on the existence of highly localised wave solutions to the vacuum Einstein equations in polarised translational symmetry. These require ancillary finite-time existence results for solutions with initial data whose amplitude and concentration make certain low Sobolev norms large. We describe a coordinate scaling extending the short-pulse ansatz of Christodoulou under which the initial amplitude becomes small, and show that existence follows from using decay obtained from a Klainerman-Sobolev inequality. We will then describe solutions which are highly spatially localised initially and remain so for finite time. We will discuss the possibility of obtaining measure-valued solutions by passing to a limit. This is joint work with Spyros Alexakis.

STEFAN CZIMEK, ICERM @ Brown University
The characteristic gluing problem of general relativity

In this talk we introduce the characteristic gluing problem for the Einstein vacuum equations. We show that the geometric obstructions to characteristic gluing of spacetimes are coming from conservation laws along null hypersurfaces. We identify these conservation laws to be the conservation of energy, linear momentum, angular momentum and the equation of motion for the center of mass. Based on this identification, we explain how to characteristically glue a given spacetime to a suitably chosen Kerr spacetime. Moreover, we describe how our characteristic gluing method yields an alternative proof of the Corvino-Schoen gluing for spacelike initial data. This is joint work with S. Aretakis (Toronto) and I. Rodnianski (Princeton).

RITA TEIXEIRA DA COSTA, Cambridge
Mode stability for extremal Kerr black holes

The Teukolsky master equations are a family of PDEs describing the linear behavior of perturbations of the Kerr black hole family, of which the wave equation is a particular case.

We prove that, for extremal Kerr black holes, the Teukolsky equations admit no exponentially growing modes nor modes on the real axis. While the result was previously known for subextremal spacetimes, we show that the proof for the latter cannot be extended to the extremal case as the nature of the event horizon changes radically in the extremal limit.

WILLIAM EAST, Perimeter Institute
Evolving Gravity Beyond Einstein

Gravitational wave observations of black hole and other compact object mergers have provided an unparalleled way to test our understanding of gravity, and have already been used to constrain a number of possible deviations from general relativity. However, despite the success of these observations, for many theories that introduce modifications to the Einstein equations, there are limited or no results on the well-posedness of the resulting initial value problem. Thus it is unclear how to, or even if one can, obtain a full theoretical prediction of what happens, e.g., when two black holes merge. I will discuss some recent progress in this regard, in particular the introduction of the modified harmonic formulation of Horndeski theories of gravity, a general class of theories of a metric coupled to a scalar field that give second order equations of motion. Using numerical solutions in this formulation, and focusing on the particular case of Einstein-scalar-Gauss-Bonnet gravity as a first application, I will demonstrate its utility in evolving strong-field data, including black hole mergers, in a regime where the deviations from general relativity are significant. I will discuss some of the remaining challenges in understanding how to evolve modifications to general relativity.

ELENA GIORGI, Princeton University
The stability of charged black holes

Black holes solutions are parametrized by their mass, spin and charge. In this talk, I will motivate why the charge of black holes adds interesting dynamics to solutions of the Einstein equation thanks to the interaction between gravitational and electromagnetic radiations. Such radiations are solutions of a system of coupled wave equations with a symmetric structure which allows to define a combined energy-momentum tensor for the system. Finally, I will show how this physical-space approach is resolutive in the most general case of Kerr-Newman black hole, where the interaction between the radiations prevents the separability in modes.

CHRISTOPH KEHLE, ETH Zurich
Diophantine approximation as Cosmic Censor for AdS black holes

I will show an intimate connection between Diophantine approximation (associated to a small divisors problem) to the behavior of linear waves on black hole interiors with negative cosmological constant $\Lambda < 0$. We explore the consequences of this for the C^0-formulation of Strong Cosmic Censorship and how its validity may change in an unexpected way according to the notion genericity imposed.

MARCUS KHURI, Stony Brook University
Lower Bounds for the Total Mass in 3-Dimensions

We provide lower bounds for the total mass of 3-dimensional initial data sets that are based on (spacetime) harmonic functions. The technique works for both the asymptotically flat and asymptotically hyperboloidal settings. These bounds are valid without the assumption of nonnegative scalar curvature or the dominant energy condition. However, if the energy condition is assumed then the result yields a new proof of the positive mass theorem.

HARI KUNDURI, Department of Mathematics and Statistics, Memorial University
Classifying toric asymptotically flat gravitational instantons

An asymptotically flat gravitational instanton is a 4d Riemannian manifold (M, g) that is complete, Ricci flat, and approaches a quotient of $\mathbb{R}^4$ with flat metric at infinity. In analogy with the classic black hole uniqueness theorem, Gibbons-Hawking and Lapedes conjectured that the two-parameter family of Kerr instantons on $\mathbb{R}^2 \times \mathbb{S}^2$ was the unique instanton invariant under a local torus action. However, Chen and Teo recently explicitly constructed a new family of such instantons on $\mathbb{CP}^2 \setminus \mathbb{S}^1$. I will discuss ongoing work on existence and uniqueness results for gravitational instantons in this class.

MATTI LASSAS, University of Helsinki
Inverse problems for Einstein's equations and other non-linear hyperbolic equations

We consider inverse problems for non-linear wave equations, for example, for the equation $\Box_g u + au^2 = f$ on a Lorentzian manifold (M, g). We study the question, do the observations of the solutions $u|_V$ on an open subset $V \subset M$, that correspond to sources f supported in V, determine the properties of the metric g in a larger domain $W \subset M$ containing V. The domain W can be the maximal domain to where the information sent from V can propagate and return back to V. In addition, we consider inverse problems for the coupled Einstein equations and matter field equations.

To study these problems we define the concept of light observation sets and show that these sets determine the conformal class of the metric.

The results have been done in collaboration with Ali Feizmohammadi, Yaroslav Kurylev, Lauri Oksanen, Gunther Uhlmann, and Yiran Wang.

ROBERT MCCANN, University of Toronto
Inscribed radius bounds for lower Ricci bounded metric measure spaces with mean convex boundary

Consider an essentially nonbranching metric measure space with the measure contraction property of Ohta and Sturm. We prove a sharp upper bound on the inscribed radius of any subset whose boundary has a suitably signed lower bound on its generalized mean curvature. This provides a nonsmooth analog of results dating back to Kasue (1983) in the Riemannian case and to Hawking (1966) in the Lorentzian case. We prove a stability statement concerning such bounds and — in the Riemannian curvature-dimension (RCD) setting — characterize the cases of equality. This represents joint work with Annegret Burtscher, Christian Ketterer and Eric Woolgar.

GEORGIOS MOSCHIDIS, UC Berkeley
The instability of Anti-de Sitter spacetime for the Einstein-scalar field system

The AdS instability conjecture provides an example of weak turbulence appearing in the dynamics of the Einstein equations in the presence of a negative cosmological constant. The conjecture claims the existence of arbitrarily small perturbations to the initial data of Anti-de Sitter spacetime which, under evolution by the vacuum Einstein equations with reflecting boundary conditions at conformal infinity, lead to the formation of black holes after sufficiently long time.

In this talk, I will present a rigorous proof of the AdS instability conjecture in the setting of the spherically symmetric Einstein-scalar field system. The construction of the unstable initial data will require carefully designing a family of initial configurations of localized matter beams and estimating the exchange of energy taking place between interacting beams over long periods of time, as well as estimating the decoherence rate of those beams. I will also discuss possible paths for extending these ideas to the vacuum case.

ACHILLEAS PORFYRIADIS, Harvard University
Extreme Black Hole Anabasis

We study the SL(2) transformation properties of spherically symmetric perturbations of the Bertotti-Robinson universe and identify an invariant μ that characterizes the backreaction of these linear solutions. The only backreaction allowed by Birkhoff's theorem is one that destroys the $AdS_2 \times S^2$ boundary and builds the exterior of an asymptotically flat Reissner-Nordstrom black hole with $Q = M\sqrt{1-\mu/4}$. We call such backreaction with boundary condition change an $anabasis$. We show that the addition of linear anabasis perturbations to Bertotti-Robinson may be thought of as a boundary condition that defines a $connected\ AdS_2 \times S^2$. The connected AdS_2 is a nearly-AdS_2 with its SL(2) broken appropriately for it to maintain connection to the asymptotically flat region of Reissner-Nordstrom. We perform a backreaction calculation with matter in the connected $AdS_2 \times S^2$ and show that it correctly captures the dynamics of the asymptotically flat black hole.

JACQUES SMULEVICI, Sorbonne Université, Laboratoire Jacques-Louis Lions
Recent results on the initial boundary value problem in GR

I will review recent results obtained in collaboration with Grigorios Fournodavlos concerning the Initial Boundary Value Problem (IBVP) for the vacuum Einstein equations. In particular, I will explain how to formulate a well-posed IBVP for the Einstein equations in the maximal gauge and, in another setting, I will present a short proof of well-posedness for the IBVP in the case of umbilic boundary.

ERIC WOOLGAR, University of Alberta
An almost splitting theorem and the topology of the Universe

Cosmic microwave background observations show that the mass density of the Universe is enough to ensure its spatial closure, with about 70% confidence. To reach this conclusion, cosmologists often rely on highly symmetric FLRW or, more generally, Bianchi models, but in fact such a restriction is not necessary. Closure follows much more generally from a Myers-type

theorem. This theorem also imposes strong topological constraints on a Universe that exceeds closure density. But there remains a 30% chance that closure density is not reached. In that case, the Universe may still be closed, and we are still able to find restrictions on the allowed topologies. Although cosmologists often consider compact hyperbolic Universes to model this scenario, an almost-splitting theorem of Cheeger-Colding type applies here and suppresses fundamental groups of polynomial growth. This is based on joint work with M Khuri and GJ Galloway.