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Directed Cycle Systems via Signed Langford Sequences

For positive integers d and t, a Langford sequence of order t and defect d is a sequence $\mathcal{L}_d^t = (s_1, \ldots, s_{2t})$ of length $2t$ that satisfies (i) for every $k \in \{d, d+1, \ldots, t+d-1\}$, there are exactly two elements $s_i, s_j \in \mathcal{L}_d^t$ such that $s_i = s_j = k$ and (ii) if $s_i = s_j = k$ with $i < j$, then $j - i = k$. Note that (ii) could be written as $j - i - k = 0$ or $i + k - j = 0$. Hence, one generalization of a Langford sequence is as follows. For positive integers d and t, a *signed* Langford sequence of order t and defect d is a sequence $\pm\mathcal{L}_d^t = (s_{-2t}, s_{-2t+1}, \ldots, s_{-1}, *, s_1, \ldots, s_{2t})$ of length $4t+1$ that satisfies (i) for every $k \in \pm\{d, d+1, \ldots t+d-1\}$, there are exactly two elements $s_i, s_j \in \pm\mathcal{L}_d^t$ such that $s_i = s_j = k$ and (ii) if $s_i = s_j = k$ with $i < 0 < j$, then $i + j + k = 0$. In this talk, we give necessary and sufficient conditions for the existence of a signed Langford sequence of order t and defect d for all positive integers d. We will then use these sequences to find cyclic decompositions of circulant digraphs into directed m-cycles for $m \geq 3$. In particular, we find a cyclic m-cycle decomposition of the complete symmetric digraph K_{2m+1}^* for all $m \geq 3$.